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Twisted backgrounds, PP-waves and nonlocal field theories

Mohsen Alishahiha
Institute for Studies in Theoretical Physics and Mathematics (IPM)
P.O. Box 19395-5531, Tehran, Iran
E-mail: alishah@theory.ipm.ac.ir

Ori J. Ganor
Department of Physics, 366 Le Conte Hall
Berkeley, CA 94720, USA, and
Lawrence Berkeley National Laboratory
Berkeley, CA 94720, USA
E-mail: origa@socrates.berkeley.edu

Abstract: We study partially supersymmetric plane-wave like deformations of string theories and M-theory on brane backgrounds. These deformations are dual to nonlocal field theories. We calculate various expectation values of configurations of closed as well as open Wilson loops and Wilson surfaces in those theories. We also discuss the manifestation of the nonlocality structure in the supergravity backgrounds. A plane-wave like deformation of little string theory has also been studied.

Keywords: Field Theories in Higher Dimensions, D-branes, Penrose limit and pp-wave background, AdS-CFT and dS-CFT Correspondence.

Contents

1. Introduction

The pp-wave metric

$$ds^2 = dx^+dx^- - \sum_{i=1}^{D-2}(dx^i)^2 - K(x_1,\dots,x_{D-2})(dx^+)^2$$

is a deformation of the $(D-1)+1$ dimensional Minkowski metric with the property that all the scalars that can be constructed out of the curvature tensor are identically zero. Realizations of such metrics in string theory have been extensively studied recently (see for example [1]–[14]). In string theory such a metric can be realized if there is also a nonzero RR or NSNS flux. For example, the following equation describes a good type-II background when the string coupling constant $g_s = 0$,

$$\begin{aligned} ds^2 &= dx^+dx^- - d\vec{x}^\top d\vec{x} - \alpha'^{-2}(\vec{x}^\top\mathcal{M}^\top\mathcal{M}\vec{x})(dx^+)^2\,, \qquad H = \eta^{-1}\alpha'^{-2}d\vec{x}^\top\wedge\mathcal{M}d\vec{x}\wedge dx^+\,,\\ \vec{x} &\stackrel{\text{def}}{=} (x^1,\dots,x^8)\,. \end{aligned} \tag{1.1}$$

Here H is a 3-form NSNS flux (for $\eta = 1$) or RR flux (for $\eta = g_s$, the string coupling constant) and $\mathcal{M}$ is a constant antisymmetric 8×8 matrix. Such backgrounds have been constructed in [7, 15, 16]. In lightcone gauge the worldsheet theories that correspond to such backgrounds are free. To see this in the case of NSNS flux requires a change of variables [13] (see also [17]), and the case of RR flux was demonstrated in [10].

One can also find a deformation of the $AdS_5\times S^5$ metric by a $(dx^+)^2$ term as follows:

$$\begin{aligned} ds^2 &= \frac{R^2}{r^2}(dx^+dx^- - d\vec{x}^\top d\vec{x} - dr^2) - \frac{R^2}{r^4}(\hat{n}^\top\mathcal{M}^\top\mathcal{M}\hat{n})(dx^+)^2 - R^2 d\hat{n}^\top d\hat{n}\,,\\ B &= \frac{1}{r^2}d\hat{n}^\top\mathcal{M}\hat{n}\wedge dx^+\,. \end{aligned} \tag{1.2}$$

Here $\hat{n}$ is a unit vector in $\mathbb{R}^6$ that parameterizes the S^5 of radius R and we have written the NSNS 2-form field potential B instead of the field strength. There is also a 5-form flux. It is also possible to replace the NSNS 2-form B in (1.2) with an RR 2-form field with exactly the same form but with a prefactor of g_s^{-1}. Backgrounds similar to those have been recently discussed in [18]. This background has the property that all the scalars that can be constructed out of the curvature tensor are identical to those of the undeformed $AdS_5\times S^5$ space. Since $AdS_5\times S^5$ is dual to $\mathcal{N}=4$ SU(N) Super-Yang-Mills theory [19] it is interesting to find out what field theory is dual to the supergravity background (1.2).[1] It would be also interesting to generalize this deformation for other brane backgrounds [22].

[1]Background similar to (1.2) has also been studied in [20] in the context of lightlike noncommutativity field theory [21].

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One of the purposes of this paper is to describe the field theory duals of (1.2) and other backgrounds that can be similarly described as a deformation of $AdS_q \times S^p$ with a $(dx^+)^2$ in the metric and with lightlike [i.e. of the form $(\cdots)\wedge dx^+$] fluxes. We will find that (1.2) describes the large N limit of a gauge theory with nonlocal interactions. We will describe how the nonlocal interactions manifest themselves in (1.2) and we will study expectation values of various configurations of Wilson loops in the theory.

Another purpose of this paper is to observe that nonlocal deformations of field theories are a rather general phenomenon that occurs when $AdS_q \times S^p$ is deformed with extra Lorentz invariance breaking fluxes. We will describe a deformation of $AdS_7 \times S^4$ that corresponds to a nonlocal deformation of the $(2,0)$ theory and the nonlocality is described by a parameter that is a 2-form. We will also discuss various deformations of the 5+1D little string theory [23, 24].

The paper is organized as follows. In sections 2–3 we present a set of backgrounds that can be obtained from flat space by a simple construction. These "twisted" backgrounds will serve as the ambient space for brane probes in section 4. We will argue that the theories on the probes are in general nonlocal. We will study their large N limit in section 5. There we will discover that the backgrounds such as (1.2) are the supergravity duals of nonlocal field theories. We follow in section 6 with a study of Wilson loops and their generalizations in such backgrounds. In section 7 we will demonstrate various manifestations of the nonlocal nature of the supergravity backgrounds. The last section is devoted to the discussion.

2. Construction of twisted backgrounds

Our goal in this section is to construct backgrounds for M-theory and string theory. These will be non-geometrical backgrounds where brane probes are, in general, described by nonlocal field theories. These backgrounds are a subset of those constructed in [7, 4, 25] and we will review them below. To construct the backgrounds we start with M-theory or string-theory compactified on T^d. We then continuously deform the geometry while keeping the space locally flat. We do this by introducing a "geometrical" twist. Using T-duality or U-duality on these backgrounds we will then construct non-geometrical twists.

2.1 Geometrical twists

We will use the term "geometrical twist" to refer to a compactification on T^d with nontrivial Wilson loops for the transverse $\mathrm{Spin}(9-d)$ (in string theory, or in M-theory $\mathrm{Spin}(10-d)$) rotation group. Take d commuting elements

$$\Omega_1, \Omega_2, \ldots, \Omega_d \in \mathrm{Spin}(9-d)$$

and define the space that is locally $\mathbb{R}^{9,1}$ but with the global identifications

$$(x_0,\, x_1, \ldots, x_d,\, \vec{x}) \sim \left(x_0,\, x_1 + 2\pi n_1 R_1, \ldots, x_d + 2\pi n_d R_d,\, \prod_{j=1}^{d} \Omega_j^{n_j} \vec{x}\right),$$
$$\vec{x} \stackrel{\text{def}}{=} (x_{d+1}, \ldots, x_9)\,. \tag{2.1}$$

A special case of this background is $\mathbb{R}^{9,1}$ with the identification

$$(x_0, x_1, \vec{x}) \sim (x_0, x_1 + 2\pi R, \Omega\vec{x}) \qquad \vec{x} \stackrel{\text{def}}{=} (x_2, \ldots, x_9)\,. \tag{2.2}$$

Here $\Omega \in \mathrm{Spin}(8)$ is a rotation matrix. We will denote this space by $\mathcal{X}(R, \Omega)$.

$$\mathcal{X}(R, \Omega) = \mathbb{R}^{9,1}/(x_0, x_1, \vec{x}) \sim (x_0, x_1 + 2\pi R, \Omega\vec{x})\,. \tag{2.3}$$

A field $\phi(x_0, x_1, \vec{x})$ in the geometry given by (2.2) can be expanded as

$$\phi(x_0, x_1, \vec{x}) = \sum_{n,\alpha} C_{n,\alpha}(x_0) e^{\frac{i(n-\omega_\alpha)x_1}{R}} Y_\alpha(\vec{x})\,.$$

Here $Y_\alpha(\vec{x})$ is an eigenfunction of Ω with eigenvalue ω_α,

$$Y_\alpha(\Omega\vec{x}) = e^{2\pi i\omega_\alpha} Y_\alpha(\vec{x})\,.$$

This expansion shows that states with a given Ω-charge have fractional momentum in the 1^{st} direction. Similarly, in the geometry (2.1) a state $|\psi\rangle$ with a specific Ω_j $(j = 1 \ldots d)$ charge given by ω_j (that is $\Omega_j|\psi\rangle = e^{2\pi i\omega_j}|\psi\rangle$) has fractional Kaluza-Klein momentum in the x_j direction and the fractional part is given by ω_j.

2.2 T-duals and U-duals of twists

We will now define a T-dual twist as follows. By definition, type-IIB on a circle of radius R with a T-dual twist given by Ω is type-IIA on $\mathcal{X}(\alpha'/R, \Omega)$ [defined in (2.3)]. Intuitively, we can think of the dual twist as a setting where a state $|\psi\rangle$ with a specific Ω charge given by ω (that is $\Omega|\psi\rangle = e^{2\pi i\omega}|\psi\rangle$) has fractional string winding number (around the circle of radius R) and the fractional part is given by ω.

In a similar fashion we can define the U-duals of a twist. We will identify the various twists according to the objects whose charges become fractional. Thus, we will call the T-dual twists F1-twists (F1 stands for the fundamental string). We can define D1-twists as the S-dual of type-IIB with an F1-twist. We can also deform type-II string theory on T^d to include Dp-twists. M-theory on T^2 can have an M2-twist parameterized by an element of the transverse rotation group $\Omega \in \mathrm{Spin}(8)$. It means that states with transverse angular momentum that transform nontrivially under Ω have a fractional wrapped M2-brane charge.

Note that even though we have used charges to characterize the twists we do not really need to single out the time direction. For example, we can define a Euclidean type-IIA theory on $\mathbb{R}^{10}$ with a D0-brane twist in $\mathrm{Spin}(10)$ as a Euclidean version of M-theory compactified on S^1 with a geometrical twist. This is the Melvin solution [1, 3]. U-duality then suggests that other Dp-brane twists are parameterized by elements in $\mathrm{Spin}(10-p)$ rather than $\mathrm{Spin}(9-p)$. Similarly, in M-theory we can have M5-brane twists parameterized by an element of $\mathrm{Spin}(6)$ [rather than $\mathrm{Spin}(5)$].

In all those cases where we have M-theory or string theory on T^p with volume V and wrapped p-brane twists $\Omega \in \mathrm{Spin}(k)$ ($k \leq 10-p$ in string theory and $\leq 11-p$ in M-theory) we can take the decompactification limit

$$V \to \infty, \quad \Omega \to I, \quad -\frac{i}{4\pi^2} V(\Omega - I) \to M = (\text{fixed}) \in so(k)\,.$$

We get a background where an eigenstate $|\psi\rangle$ of M [acting as an $so(k)$ angular momentum generator] that satisfies $M|\psi\rangle = \mathcal{U}|\psi\rangle$ formally has the charge of a piece of a p-brane with volume $2\pi\mathcal{U}$. (The 2π factors will simplify upcoming formulas.)

In this way we can obtain backgrounds that are parameterized by an element $M \in so(k)$ and preserve translational invariance in $10-k$ (in string theory, or $11-k$ in M-theory) directions.

2.3 Notation

We will start from type-II toroidal compactifications where space-time is of the form $\mathbb{R}^{1,d} \times S^1 \times \cdots \times S^1$. We use the following notation for charges:

$\mathcal{Q}\mathrm{D}_{[]}$	the charge of a D0-brane.
$\mathcal{Q}\mathrm{D}_{[I_1 I_2 \dots I_p]}$	the charge of a Dp-brane that wraps directions $I_1 \dots I_p$.
$\mathcal{Q}\mathrm{KK}_{[I]}$	the charge of a Kaluza-Klein particle in direction I.
$\mathcal{Q}\mathrm{FS}_{[I]}$	the charge of a fundamental string wrapping direction I.
$\mathcal{Q}\mathrm{NS5}_{[I_1 \dots I_5]}$	the charge of an NS5-brane wrapping directions $I_1 \dots I_5$.
$\mathcal{Q}\mathrm{M2}_{[I_1 I_2]}$	the charge of an M2-brane in directions $I_1 I_2$.
$\mathcal{Q}\mathrm{M5}_{[I_1 \dots I_5]}$	the charge of an M5-brane in directions $I_1 \dots I_5$.

If i_1, i_2 are noncompact directions, we will denote by $J_{[i_1 i_2]}$ the angular momentum that corresponds to rotations in the $i_1 - i_2$ plane. $J_{[i_1 i_2]}$ is therefore integral for bosons and half-integral for fermions. A geometrical twist such as (2.2) with Ω a rotation in the $2-3$ plane by an angle $2\pi\alpha$, for example, will be described by the equation:

$$\mathcal{Q}\mathrm{KK}_{[1]} - \alpha J_{[2,3]} \in \mathbb{Z}\,.$$

We define

$$\begin{aligned}
&\mathcal{V}\mathrm{D}_{[I_1 I_2 \dots I_p]} \stackrel{\mathrm{def}}{=} (2\pi)^p R_{I_1} \cdots R_{I_p}\ \mathcal{Q}\mathrm{D}_{[I_1 I_2 \dots I_p]}\,, \qquad && \mathcal{V}\mathrm{FS}_{[I]} \stackrel{\mathrm{def}}{=} 2\pi R_I\ \mathcal{Q}\mathrm{FS}_{[I]}\,,\\
&\mathcal{V}\mathrm{M2}_{[I_1 I_2]} \stackrel{\mathrm{def}}{=} (2\pi)^2 R_{I_1} R_{I_2}\ \mathcal{Q}\mathrm{M2}_{[I_1 I_2]}\,, && \mathcal{V}\mathrm{M5}_{[I_1 \dots I_5]} \stackrel{\mathrm{def}}{=} (2\pi)^5 R_{I_1} \cdots R_{I_5}\ \mathcal{Q}\mathrm{M5}_{[I_1 \dots I_5]}\\
&\mathcal{V}\mathrm{NS5}_{[I_1 \dots I_5]} \stackrel{\mathrm{def}}{=} (2\pi)^5 R_{I_1} \cdots R_{I_5}\ \mathcal{Q}\mathrm{NS5}_{[I_1 \dots I_5]}\,.
\end{aligned} \tag{2.4}$$

The prefactors are the volumes of the corresponding branes.

3. Supergravity solutions of generalized twisted backgrounds

In this section we shall study the general twisted background in type II string theories and M-theory. These can be obtained from compactification of string/M-theory on a torus with nontrivial Wilson loops for the transverse rotation group as we demonstrated in the previous section. Probing this background with a brane would lead to a nonlocal field theory. One can also make a boost in the obtained backgrounds to find backgrounds with lightlike twist (see [26]). Probing this background will also give a field theory with nonlocality in a lightlike direction. A special case of a D3-brane has recently been studied in [27] where a (nonlocal) dipole theory is obtained. It is the aim of this section to generalize this construction for other brane backgrounds in string/M-theory.

3.1 Fundamental string twist

Concentrating on the geometry of (2.2) we take the limit

$$R \to 0\,, \qquad \Omega \to I\,, \qquad R^{-1}(\Omega - I) \to 2\pi i {\alpha'}^{-1} M = \text{fixed}\,.$$

Here M is an $so(8)$ lie-algebra valued element with dimensions of length. Using T-duality, the metric (in string units) is found to be

$$ds^2 = dt^2 - \frac{1}{1 + {\alpha'}^{-2}\vec{x}^\top M^\top M\vec{x}} dx_1^2 - d\vec{x}^\top d\vec{x} + \frac{(d\vec{x}^\top M\vec{x})^2}{\alpha'^2 + \vec{x}^\top M^\top M\vec{x}}\,. \tag{3.1}$$

Here $\vec{x}$ denotes the coordinate in the 8 transverse directions and $0 \le x_1 \le 2\pi$. We also have an NSNS 2-form and a dilaton

$$B = \frac{d\vec{x}^\top M\vec{x}}{\alpha'^2 + \vec{x}^\top M^\top M\vec{x}} \wedge dx^1\,, \qquad e^{2(\phi - \phi_0)} = \frac{1}{1 + {\alpha'}^{-2}\vec{x}^\top M^\top M\vec{x}}\,. \tag{3.2}$$

This is the background that we denoted by

$$\mathcal{V}\mathrm{FS}_{[1]} - 2\pi M_{ij} J_{[ij]} \in \mathbb{Z}\,.$$

Note that the dilaton can be made small everywhere. The metric, however, becomes singular as $|\vec{x}| \to \infty$ and therefore α'-corrections are important when $M|\vec{x}| \sim \alpha'$.

Similarly one can find a twisted background in which M takes its value in the $so(9-p)$ Lie-algebra. In this case the metric is given by

$$ds^2 = dt^2 - \frac{1}{1 + {\alpha'}^{-2}\vec{x}^\top M^\top M\vec{x}} dx_p^2 - \sum_{i=1}^{p-1} dx_i^2 - d\vec{x}^\top d\vec{x} + \frac{(d\vec{x}^\top M\vec{x})^2}{\alpha'^2 + \vec{x}^\top M^\top M\vec{x}}\,,$$

while the B field and the dilaton are the same as (3.2).

3.2 Generalized twists

The S-dual configuration to (3.1)-(3.2) is given by

$$\begin{aligned} ds^2 &= e^{\phi_0}(1 + {\alpha'}^{-2}\vec{x}^\top M^\top M\vec{x})^{1/2}\left[dt^2 - d\vec{x}^\top d\vec{x}\right] - \\ &\quad - e^{\phi_0}(1 + {\alpha'}^{-2}\vec{x}^\top M^\top M\vec{x})^{-1/2}\left[dx_1^2 - {\alpha'}^{-2}(d\vec{x}^\top M\vec{x})^2\right]\,, \end{aligned} \tag{3.3}$$

$$C^{(RR)} = \frac{d\vec{x}^\top M\vec{x}}{\alpha'^2 + \vec{x}^\top M^\top M\vec{x}} \wedge dx^1\,, \qquad e^{\phi} = e^{\phi_0}(1 + {\alpha'}^{-2}\vec{x}^\top M^\top M\vec{x})^{1/2}\,. \tag{3.4}$$

This is the background that we denoted by

$$\mathcal{V}\mathrm{D}_{[1]} - 2\pi M_{ij} J_{[ij]} \in \mathbb{Z}\,.$$

Here $M \in so(8)$ [or even $so(8,1)$] but if we take $p > 1$ and restrict M to an $so(9-p)$ subgroup [or perhaps $so(9-p,1)$] we can compactify $(p-1)$ directions and apply T-duality to get the type-II metric

$$
\begin{aligned}
ds^2 &= e^{\phi_0}(1+\alpha'^{-(p+1)}\vec{x}^\top M^\top M\vec{x})^{1/2}\left[dt^2 - d\vec{x}^\top d\vec{x}\right] - \\
&\quad - e^{\phi_0}(1+\alpha'^{-(p+1)}\vec{x}^\top M^\top M\vec{x})^{-1/2}\left[\sum_{i=1}^{p} dx_i^2 - \alpha'^{-(p+1)}(d\vec{x}^\top M\vec{x})^2\right], \qquad (3.5) \\
C^{(RR)} &= e^{\frac{p-1}{2}\phi_0}\frac{d\vec{x}^\top M\vec{x}}{\alpha'^{p+1}+\vec{x}^\top M^\top M\vec{x}}\wedge dx^1\wedge\cdots\wedge dx^p, \quad e^{\phi} = \frac{e^{\phi_0}}{(1+\alpha'^{-(p+1)}\vec{x}^\top M^\top M\vec{x})^{\frac{p-3}{4}}}. \qquad (3.6)
\end{aligned}
$$

In the above formula we have absorbed a factor of $\alpha'^{\frac{p-1}{2}}$ in M so as to make it of dimensions $[\text{length}]^p$. In the notation of 2.3 this background corresponds to

$$\mathcal{V}\mathrm{D}_{[1\ldots p]} - 2\pi M_{ij}J_{[ij]} \in \mathbb{Z}\,.$$

We can lift the type-IIA metric with a D2-brane twist to obtain M-theory with an M2-twist.

$$
\begin{aligned}
ds^2 &= (1+l_p^{-6}\vec{x}^\top M^\top M\vec{x})^{1/3}\left[dt^2 - d\vec{x}^\top d\vec{x} - dx_{10}^2\right] - \\
&\quad -(1+l_p^{-6}\vec{x}^\top M^\top M\vec{x})^{-2/3}\left[\sum_{i=1}^{2} dx_i^2 - l_p^{-6}(d\vec{x}^\top M\vec{x})^2\right], \qquad (3.7) \\
C &= \frac{d\vec{x}^\top M\vec{x}}{l_p^6+\vec{x}^\top M^\top M\vec{x}}\wedge dx^1\wedge dx^2\,. \qquad (3.8)
\end{aligned}
$$

In the notation of 2.3 this background corresponds to

$$\mathcal{V}\mathrm{M2}_{[12]} - 2\pi M_{ij}J_{[ij]} \in \mathbb{Z}\,.$$

We can also lift the type-IIA metric with a D4-brane twist to obtain M-theory with an M5-twist.

$$
\begin{aligned}
ds^2 &= (1+l_p^{-12}\vec{x}^\top M^\top M\vec{x})^{2/3}\left[dt^2 - d\vec{x}^\top d\vec{x}\right] - \\
&\quad -(1+l_p^{-12}\vec{x}^\top M^\top M\vec{x})^{-1/3}\left[\sum_{i=1}^{4} dx_i^2 + dx_{10}^2 - l_p^{-12}(d\vec{x}^\top M\vec{x})^2\right], \qquad (3.9) \\
{}^*dC &= \left[\frac{d\vec{x}^\top\wedge Md\vec{x}}{l_p^{12}+\vec{x}^\top M^\top M\vec{x}} - 2\frac{d\vec{x}^\top M\vec{x}\wedge d\vec{x}^\top M^\top M\vec{x}}{(l_p^{12}+\vec{x}^\top M^\top M\vec{x})^2}\right]\wedge dx^1\wedge\cdots\wedge dx^4\wedge dx^{10}\,. \qquad (3.10)
\end{aligned}
$$

In the notation of 2.3 this background corresponds to

$$\mathcal{V}\mathrm{M5}_{[1,\ldots,4,10]} - M_{ij}J_{[ij]} \in \mathbb{Z}\,.$$

Finally we can lift the type-IIA metric with a D6-brane twist. In this case $\vec{x}$ is 3-dimensional and we can take M to be proportional to the generator of rotations in directions $8, 9$. Let

${\alpha'}^{7/2}m$ denote its magnitude. We also set $z \stackrel{\text{def}}{=} x_8 + ix_9$. Thus,

$$
\begin{aligned}
&1 + {\alpha'}^{-(p+1)}\vec{x}^\top M^\top M\vec{x} = 1 + m^2|z|^2\,, \qquad d\vec{x}^\top M\vec{x} = \frac{im}{2}{\alpha'}^{7/2}(zd\overline{z} - \overline{z}dz)\,,\\
ds^2 &= e^{\phi_0}(1+m^2|z|^2)^{1/2}\left[dt^2 - |dz|^2 - dx_7^2\right] -\\
&- e^{\phi_0}(1+m^2|z|^2)^{-1/2}\left[\sum_{i=1}^6 dx_i^2 - \frac{m^2}{4}|zd\overline{z} - \overline{z}dz|^2\right]\,, \qquad (3.11)\\
dC^{(RR)} &= \frac{i{\alpha'}^{-7/2}e^{5/2\phi_0}m}{(1+m^2|z|^2)^2}dz\wedge d\overline{z}\wedge dx^1\wedge\cdots\wedge dx^6\,, \quad e^\phi = e^{\phi_0}(1+m^2|z|^2)^{-3/4}\,. \qquad (3.12)
\end{aligned}
$$

Note that

$$\sqrt{-g} = (1+m^2|z|^2)^{-1}\,.$$

The dual RR field is

$${}^*dC^{(RR)} = l_p^{-1}m dt\wedge dx^7\,.$$

Thus the 1-form RR-field can be taken to be

$$A^{(RR)} = -l_p^{-1}mx^7dt\,.$$

Lifting to M-theory and setting $z = re^{i\theta}$ we obtain the metric

$$ds^2 = (1+m^2r^2)\left[dt^2 - dr^2 - dx_7^2\right] - r^2d\theta^2 - \frac{1}{1+m^2r^2}(dx_{10} + mx_7dt)^2 - \sum_{i=1}^6 dx_i^2 \quad (3.13)$$

and no fluxes. But note that it is not supersymmetric and the issue of stability is not clear.

3.3 Lightlike twists

We can also find lightlike twisted backgrounds of string theory or M-theory by making use of a boost from the dipole twisted backgrounds which have been described so far (see also [25]). This can also be thought of as taking the Penrose limit of the corresponding dipole twisted backgrounds. To begin with we consider the following boost in the x_p direction

$$\hat{t} = \cosh\gamma\, t - \sinh\gamma\, x_p\,, \qquad \hat{x}_p = -\sinh\gamma\, t - \cosh\gamma\, x_p\,, \qquad (3.14)$$

or

$$x^+ = e^{-\gamma}y^+\,, \qquad x^- = e^{\gamma}y^-\,, \qquad (3.15)$$

with $y^\pm = x_p \pm t$ and $x^\pm = \hat{x}_p \pm \hat{t}$.

To have a lightlike dipole we now take the infinite boost limit, $\gamma \to \infty$. In order to end up with a lightlike dipole vector with finite component we must simultaneously scale $M \to 0$ while keeping the following quantity fixed

$$\widetilde{M} \stackrel{\text{def}}{=} Me^{\gamma} = \text{finite}\,. \qquad (3.16)$$

In this limit the background (3.6) reads

$$\begin{aligned} ds^2 &= dx^+dx^- + \frac{1}{4}\alpha'^{-(p+1)}(\vec{x}^\top \widetilde{M}^\top \widetilde{M}\vec{x})(dx^+)^2 - d\vec{x}^\top d\vec{x} - \sum_{i=1}^{p-1}(dx^i)^2\,, \\ C^{RR} &= \frac{1}{2}\alpha'^{-(p+1)}(d\vec{x}^\top \widetilde{M}\vec{x})\wedge dx^1\wedge\cdots\wedge dx^{p-1}\wedge dx^+, \qquad e^{2(\phi-\phi_0)} = 1\,, \end{aligned} \tag{3.17}$$

which is the RR pp-wave specified by

$$\mathcal{V}\mathrm{D}_{[1,\cdots,(p-1),+]} - 2\pi\widetilde{M}_{ij}J_{[ij]} \in \mathbb{Z}\,.$$

Similarly we can also find the NSNS pp-wave background by making use of a boost from the twisted backgrounds studied in subsection 3.1. For example the lightlike background specified by

$$\mathcal{V}\mathrm{FS}_{[+]} - 2\pi\widetilde{M}_{ij}J_{[ij]} \in \mathbb{Z}\,,$$

is the NSNS pp-wave given by

$$\begin{aligned} ds^2 &= dx^+dx^- + \frac{1}{4}\,\alpha'^{-2}(\vec{x}^\top \widetilde{M}^\top \widetilde{M}\vec{x})(dx^+)^2 - d\vec{x}^\top d\vec{x}\,, \\ B &= \frac{1}{2}\,\alpha'^{-2}(d\vec{x}^\top \widetilde{M}\vec{x})\wedge dx^+\,, \qquad e^{2(\phi-\phi_0)} = 1\,. \end{aligned} \tag{3.18}$$

The same procedure can be applied to M-theory twisted backgrounds. In this way we will be able to find the lightlike twisted background in M-theory. For example from the M2-twist background (3.7) of M-theory using a boost similar to (3.14) and (3.15) one finds

$$\begin{aligned} ds^2 &= dx^+dx^- + \frac{1}{4}\,l_p^{-6}(\vec{x}^\top \widetilde{M}^\top \widetilde{M}\vec{x})(dx^+)^2 - d\vec{x}^\top d\vec{x} - dx_1^2 - dx_{10}^2\,, \\ C &= \frac{1}{2}\,l_p^{-6}(d\vec{x}^\top \widetilde{M}\vec{x})\wedge dx^1\wedge dx^+\,, \end{aligned} \tag{3.19}$$

which corresponds to

$$\mathcal{V}\mathrm{M2}_{[1+]} - 2\pi\widetilde{M}_{ij}J_{[ij]} \in \mathbb{Z}\,.$$

The lightlike twist background of M-theory corresponding to $\mathcal{V}\mathrm{M5}_{[1,\cdots,4,+]} - 2\pi\widetilde{M}_{ij}J_{[ij]} \in \mathbb{Z}$ can also be obtained from the Penrose limit of the M5-twist background.

We note that the pp-wave backgrounds studied in this section provide string theory backgrounds in which the string theory can be exactly solved. The lightlike twist will also provide pp-wave backgrounds of M-theory and it would be interesting to study the Matrix model of these pp-wave backgrounds. In particular one can find a pp-wave-like background in M-theory without any fluxes. This background can be obtained by taking the Penrose limit from (3.13)

$$ds^2 = dx^+dx^- + \frac{1}{4}\widetilde{M}^2|z|^2(dx^+)^2 - |dz|^2 - dx_7^2 - \sum_{i=1}^{5}dx_i^2 - \left(dx_{10} + \frac{1}{2}\widetilde{M}x^7dx^+\right)^2\,. \tag{3.20}$$

We note, however, that it is not obvious whether this is a stable background of M-theory. Backgrounds somewhat reminiscent of this have recently been discussed in [28]–[30].

4. Probing with branes and new nonlocal theories

We will now examine brane probes in the various twisted geometries. In many cases we will discover new types of nonlocal field theories. The configurations that we will discuss are related to configurations of D-branes in pp-wave backgrounds. Such configurations have been discussed extensively in [17] and [31]–[40] but mostly in the context of the pp-wave limit relevant for $AdS_5 \times S^5$ [14] and not so much for the twisted backgrounds.

4.1 Fundamental string twists

In this case $p = 1$ and M has dimensions of length. We take the fundamental string twist to be in the direction of x_1 and take N Dd-branes that extend in directions $0, 1 \ldots d$. The twist M is then an element of the Lie algebra $so(9-d)$ corresponding to rotations in the remaining directions. This case of a fundamental string twist has been studied [41, 42] and in the case that $M \gg \alpha'^{1/2}$ the resulting low-energy description of the dynamics of the brane is a theory of fundamental dipoles with lengths proportional to the eigenvalues of $2\pi M$. In order to be self-contained we have included a review of dipole theories in appendix. The proof that the dipole-theory is indeed the low energy description on the brane probe will not be repeated here, but intuitively we can argue as follows. On a D-brane the fundamental string is identified with electric flux. So dipoles naturally behave like fixed size open strings. The statement that states with $\mathrm{Spin}(9-d)$ charge also have a finite string length translates on the D-branes to the statement that states with R-symmetry charge are dipoles.

If, on the other hand, the D-brane probes are transverse to the twist we get a massive deformation of Super Yang-Mills theory. For example, let us take N D$(d-1)$-probes in directions $0, 2, \ldots, d$. It is convenient to compactify the direction of x_1 on a circle of radius $R \gg |M|$. T-duality on that circle will give us Dd-branes wrapped on a circle of radius α'/R with a geometrical twist of magnitude M/R. It is not hard to see that the effect of the twist on the low energy description in d dimensions is to add a mass term to the scalars and fermions of d dimensional $\mathrm{U}(N)$ Super Yang-Mills theory. The mass term is given by

$$4\pi^2\alpha'^{-2}\phi^\top M^\top M\phi + 2\pi\alpha'^{-1}\psi^\top M\psi\,. \tag{4.1}$$

Here ϕ represents the scalars, written as a vector in the fundamental representation of $so(9-d)$ and ψ represents the fermions, written as a vector in the spinor representation of $so(9-d)$. M that appears in the formula above should be interpreted as a matrix in the appropriate representation.

Again, we can heuristically understand the mass term as follows. In the presence of the twist, particles with R-symmetry charge behave like finite fundamental strings that are perpendicular to the D-branes and have length proportional to the corresponding eigenvalue of $2\pi M$. Their mass is therefore proportional to $2\pi\alpha'^{-1}M$. Of course, in string theory there are no finite fundamental strings perpendicular to the D-brane but the intuitive picture gives the correct mass for the R-symmetry charged particles. Using the supergravity solution (3.1)–(3.2) the mass term can be interpreted as a gravitational potential that attracts the D-brane probe to the origin of the transverse space $\mathbb{R}^{9-d}$ [43].

4.2 D-brane twists

We get new nonlocal field theories when we probe the *generalized* twisted backgrounds with D-branes. For most of these cases, we do not have a simple field theory description. Let us explore various new possibilities that arise.

D3-probes with a longitudinal D1-twist. This is the S-dual theory to the D3-probes with a longitudinal fundamental string twist. Since the latter is described by the dipole-theory the S-dual should be a field theory with fundamental magnetic dipoles. For a small dipole vector matrix M the electric dipole theories can be described [44] as a deformation of $\mathcal{N}=4$ U(N) SYM by the operator

$$2\pi M^{\mu}_{IJ}\mathcal{O}^{IJ}_{\mu}, \qquad (I,J=1\dots6), \qquad (\mu=0\dots3),$$

where $\mathcal{O}^{IJ}_{\mu}$ is the dimension-5 operator

$$\mathcal{O}^{IJ}_{\mu} = \frac{i}{g^2_{\rm YM}}\mathrm{tr}\left\{F_{\mu}{}^{\nu}\Phi^{[I}D_{\nu}\Phi^{J]} + \sum_K (D_\mu\Phi^K)\Phi^{[K}\Phi^I\Phi^{J]}\right\} + \text{fermions}. \tag{4.2}$$

Here $I,J=1\dots6$ are R-symmetry indices, Φ^I $(I=1\dots6)$ are the scalars, $D_\mu=\partial_\mu - i[A_\mu,\cdot]$ is the covariant derivative, $F_{\mu\nu}$ is the field strength and $[\cdots]$ means complete anti-symmetrization. $\mathcal{O}^{IJ}_{\mu}$ is a vector operator that transforms in the **15** of the R-symmetry group SU(4).

The magnetic dipole theory should be described, to linear order in M, by the deformation $2\pi M^{\mu}_{IJ}\widetilde{\mathcal{O}}^{IJ}_{\mu}$ where $\widetilde{\mathcal{O}}^{IJ}_{\mu}$ is the dual dimension-5 operator

$$\widetilde{\mathcal{O}}^{IJ}_{\mu} = \frac{i}{g^2_{\rm YM}}\mathrm{tr}\left\{\widetilde{F}^{\nu}_{\mu}\Phi^{[I}D_{\nu}\Phi^{J]} + \sum_K (D_\mu\Phi^K)\Phi^{[K}\Phi^I\Phi^{J]}\right\} + \text{fermions}, \tag{4.3}$$

where $\widetilde{F}_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\sigma\tau}F^{\sigma\tau}$ is the dual (nonabelian) field strength.

5. Supergravity solutions of D-branes and NS5-branes and the large N limit

In this section we shall first review Dp-brane probes of the F1 twist geometry. The obtained supergravity solution is a Dp-brane solution in the presence of B field with one leg along the worldvolume and the other along the directions transverse to the brane. This background would provide the supergravity description of noncommutative dipole field theory.[2] We then consider a new set of nonlocal theories by performing a boost along the worldvolume direction of the brane in which the B field is nonzero. Generalizations to NS5-branes with RR field and M-theory with C form field will also be studied.

We note also that making a boost (the Penrose limit) in a nonlocal theory has recently been considered in [53]–[56].

[2]Special cases of dipole theories have been discussed in [45]–[47] and various aspects of the theories have been explored in [48]–[52], see also [42].

5.1 Review of the supergravity dual of dipole theory

By probing the F1 twist geometry with Dp-branes we find a supergravity solution of Dp-branes in the presence of a B-field with one leg along the worldvolume and the other along the transverse directions to the brane [42]

$$
\begin{aligned}
ds^2 &= f^{-1/2}\left(dt^2 - dx_1^2 - \cdots - dx_{p-1}^2 - \frac{\alpha'^2 dx_p^2}{\alpha'^2 + r^2 \hat{n}^\top M^\top M \hat{n}}\right) - \\
&\quad - f^{1/2}\left(dr^2 + r^2 d\hat{n}^\top d\hat{n} - \frac{r^4(\hat{n}^\top M^\top d\hat{n})^2}{\alpha'^2 + r^2 \hat{n}^\top M^\top M \hat{n}}\right), \\
e^{2\phi} &= \frac{\alpha'^2 g_s^2 f^{\frac{3-p}{2}}}{\alpha'^2 + r^2 \hat{n}^\top M^\top M \hat{n}}, \\
\sum_{a=p+1}^{9} B_{pa} dx_a &= \frac{r^2 d\hat{n}^\top M \hat{n}}{\alpha'^2 + r^2 \hat{n}^\top M^\top M \hat{n}}, \qquad (5.1)
\end{aligned}
$$

where $\hat{n}$ is an $(8-p)$-dimensional unit vector, $|\hat{n}|^2 = 1$, and

$$
f = 1 + \frac{(4\pi)^{\frac{5-p}{2}} \Gamma\left((7-p)/2\right) N g_s \alpha'^{\frac{7-p}{2}}}{r^{7-p}}.
$$

Also, M is the same $so(8-p)$ element from subsection 3.1. In general this background breaks the supersymmetry completely, but for special cases of the matrix M some supersymmetries can be left intact. Another problem that has to be considered is the stability of the solution. It is not obvious that the solutions we found are stable. Nevertheless, taking the matrix M such that the solutions preserve some amount of supersymmetries, as we will consider, would hopefully lead to stable solutions. In fact, in this paper we shall mostly consider matrices M such that 8 supersymmetries are preserved.

It has also been argued [42] that the worldvolume theory of the supergravity solution (5.1) decouples from bulk gravity leading to a nonlocal theory, i.e. noncommutative dipole theory (reviewed in appendix). The decoupling limit, in which the worldvolume theory decouples from the bulk, is defined by $\alpha' \to 0$ keeping the following quantities fixed

$$
u \stackrel{\text{def}}{=} \frac{r}{\alpha'}, \qquad \bar{g}_s \stackrel{\text{def}}{=} \alpha'^{\frac{p-3}{2}} g_s\,. \qquad (5.2)
$$

In this limit the supergravity solution (5.1) reads

$$
\begin{aligned}
\alpha'^{-1} ds^2 &= \left(\frac{u}{R}\right)^{\frac{7-p}{2}}\left(dt^2 - dx_1^2 - \cdots - \frac{dx_p^2}{1 + u^2 \hat{n}^\top M^\top M \hat{n}}\right) - \\
&\quad - \left(\frac{R}{u}\right)^{\frac{7-p}{2}}\left(du^2 + u^2 d\hat{n}^\top d\hat{n} - \frac{u^4(\hat{n}^\top M^\top d\hat{n})^2}{1 + u^2 \hat{n}^\top M^\top M \hat{n}}\right), \\
e^{2\phi} &= \bar{g}_s^2 \frac{(R/u)^{(7-p)(3-p)/2}}{1 + u^2 \hat{n}^\top M^\top M \hat{n}}, \\
\sum_{a=p+1}^{9} B_{pa} d\hat{n}_a &= \frac{u^2 d\hat{n}^\top M \hat{n}}{1 + u^2 \hat{n}^\top M^\top M \hat{n}}, \qquad (5.3)
\end{aligned}
$$

with

$$R^{7-p} = 2^{7-2p}\pi^{\frac{9-3p}{2}}\Gamma\left(\frac{7-p}{2}\right) g_{\rm YM}^2 N\,, \qquad g_{\rm YM}^2 = (2\pi)^{p-2}\bar{g}_s\,. \tag{5.4}$$

This supergravity solution provides the gravity dual of the noncommutative dipole gauge theory.

Starting from the case of $p=5$ and applying S-duality one can find the supergravity solution of type IIB NS5-branes in the presence of an RR 2-form potential with one leg along the brane and the other along the transverse directions. This could provide the gravity dual of the dipole deformation of little string theory. We can also make a series of T-duality transformations to produce a new supergravity solution. This supergravity solution describes type II NS5-branes in the presence of RR $(6-p)$-form, for $p = 0\dots 4$, with one leg along the transverse directions and $(5-p)$ legs along the NS5-branes worldvolume. The corresponding supergravity solution in the decoupling limit is given by

$$\begin{aligned} ds^2 &= (1+u^2L^2)^{1/2}\Bigg[dt^2 - \sum_{i=1}^{p} dx_i^2 - \frac{\sum_{j=p+1}^5 dx_j^2}{1+u^2L^2} - \frac{N\alpha'}{u^2}\times \\ &\qquad\qquad \times\left(du^2+u^2d\Omega_3-\frac{u^4L^2}{1+u^2L^2}(a_1d\theta_1+a_2d\theta_2+a_3d\theta_3)^2\right)\Bigg]\,, \\ e^{2\phi} &= \frac{N}{l_s^2u^2}(1+u^2L^2)^{(p-2)/2}\,, \\ \sum_{a=6}^{9} C_{(p+1)\cdots 5\theta_a}d\theta_a &= \frac{u^2L}{1+u^2L^2}(a_1d\theta_1+a_2d\theta_2+a_3d\theta_3)\,, \end{aligned} \tag{5.5}$$

where θ_i's are angular coordinates parameterizing the sphere S^3 transverse to the NS5-branes, and

$$a_1 \stackrel{\rm def}{=} \cos\theta_2\,, \qquad a_2 \stackrel{\rm def}{=} -\sin\theta_1\cos\theta_1\sin\theta_2\,, \qquad a_3 \stackrel{\rm def}{=} \sin^2\theta_1\sin^2\theta_2\,. \tag{5.6}$$

Note that in the above solution we have chosen the matrix M in such a way that the system is maximally supersymmetric which means that the solution preserves 8 supercharges. For this case the matrix M has the following form

$$M = \begin{pmatrix} 0 & L & 0 & 0 \\ -L & 0 & 0 & 0 \\ 0 & 0 & 0 & L \\ 0 & 0 & -L & 0 \end{pmatrix}. \tag{5.7}$$

5.2 Lightlike dipole theory

In this section we shall study the lightlike dipole theory using its gravity description. To find the corresponding supergravity solution we start with a background in which the probe Dp-branes have a small spacelike dipole vector and then we perform a large boost. In fact we will consider the following boost in the x_p direction

$$\hat{t} = \cosh\gamma\, t - \sinh\gamma\, x_p\,, \qquad \hat{x}_p = -\sinh\gamma\, t - \cosh\gamma\, x_p\,, \tag{5.8}$$

or

$$x^+ = e^{-\gamma} y^+ \,, \qquad x^- = e^{\gamma} y^- \,, \tag{5.9}$$

with $y^\pm = x_p \pm t$ and $x^\pm = \hat{x}_p \pm \hat{t}$.

To have a lightlike dipole we now take the infinite boost limit, $\gamma \to \infty$. In order to end up with a lightlike dipole vector with finite component we must, at the same time, also take the limit $M \to 0$ while keeping the following quantity fixed

$$\mathcal{M} \stackrel{\text{def}}{=} M e^{\gamma} = \text{finite}\,. \tag{5.10}$$

In this limit the background (5.3) reads

$$\begin{aligned} \frac{ds^2}{l_s^2} &= \left(\frac{u}{R}\right)^{\frac{(7-p)}{2}} \left[-dx^+dx^- + \frac{\hat{n}^\top \mathcal{M}^\top \mathcal{M} \hat{n}}{4}\, u^2 (dx^+)^2 - dx_1^2 - \cdots - dx_{p-1}^2\right] - \\ &\quad - \left(\frac{R}{u}\right)^{\frac{7-p}{2}} \left[du^2 + u^2 d\Omega_{8-p}^2\right], \\ e^{2\phi} &= \bar{g}_s^2 \left(\frac{R}{u}\right)^{\frac{(7-p)(3-p)}{2}}, \end{aligned} \tag{5.11}$$

and

$$\sum_{a=p+1}^{9} B_{+a} d\hat{n}_a = \frac{1}{2} u^2 d\hat{n}^\top \mathcal{M} \hat{n}\,. \tag{5.12}$$

One can now choose the matrix M such that the solution preserves 8 supercharges. For this Dp-brane case such a matrix M is given by

$$\mathcal{M} = L(e_{6-p,7-p} - e_{7-p,6-p} + e_{8-p,9-p} - e_{9-p,8-p})\,, \tag{5.13}$$

where e_{ij} are a set of $(9-p)^2$ matrices of dimensions $(9-p) \times (9-p)$ and are defined by $(e_{ij})_{kl} = \delta_{ik}\delta_{jl}$. The effective dipole moment of the noncommutative dipole theory described by (5.3) is defined to be

$$2\pi L_{\text{eff}} \stackrel{\text{def}}{=} 2\pi L \prod_{i=1}^{5-p} \sin\theta_i\,, \qquad \text{for } p = 1, \ldots, 5\,, \tag{5.14}$$

where θ_j $[j = 1 \ldots (8-p)]$ are angular coordinates parameterizing the sphere $S^{(8-p)}$ transverse to the Dp-brane such that

$$\hat{n} = \left(\cos\theta_1,\ \sin\theta_1\cos\theta_2,\ \sin\theta_1\sin\theta_2\cos\theta_3,\ \cdots,\ \prod_{j=1}^{8-p}\sin\theta_j\right).$$

In the large boost limit we have $L \to 0$ while $e^{\gamma} L$ is kept fixed. We define

$$2\pi\mu_{\text{eff}}(\theta_1, \ldots, \theta_{8-p}) \stackrel{\text{def}}{=} 2\pi e^{\gamma} L_{\text{eff}} \tag{5.15}$$

to be the finite effective magnitude of the lightlike dipole vector.

For this case with 8 preserved supercharges the metric (5.11) reads

$$\alpha'^{-1} ds^2 = \left(\frac{u}{R}\right)^{(7-p)/2} \left[-dx^+ dx^- + \frac{\mu_{\rm eff}{}^2}{4}\, u^2 (dx^+)^2 - dx_1^2 - \cdots - dx_{p-1}^2\right] - \\ - \left(\frac{R}{u}\right)^{(7-p)/2} \left[du^2 + u^2 d\Omega_{8-p}^2\right] . \qquad (5.16)$$

5.3 Deformations of little string theory

One can also proceed to study the lightlike deformation of little string theory. To do this, we start from the supergravity solution (5.5) and perform a boost in the x_5 direction. Using a boost similar to (5.8)-(5.9) in the limit $\gamma \to \infty$ and taking into account the finiteness condition (5.10) for the lightlike dipole the supergravity solution (5.5) reads

$$\begin{aligned} ds^2 &= -dx^+ dx^- + \frac{\mu^2}{4} u^2 (dx^+)^2 - N\alpha' \frac{du^2}{u^2} - N\alpha' d\Omega_3^2 - \sum_{i=1}^{4} dx_i^2 \,, \\ e^{2\phi} &= \frac{N}{\alpha' u^2} \,, \qquad \sum_a C_{(p+1)\cdots 4+\theta_a} d\theta_a = \frac{\mu u^2}{2} (a_1 d\theta_1 + a_2 d\theta_2 + a_3 d\theta_3) \,, \end{aligned} \qquad (5.17)$$

where $\mu = e^\gamma L$ is again the magnitude of the lightlike dipole vector. Note that there is also a two form B field representing the charge of the NS5-branes which is given by

$$dB = \alpha' N \epsilon_3 \,, \qquad (5.18)$$

where ϵ_3 is the volume form of the S^3 sphere transverse to the NS5-branes.

Interestingly enough, the string theory in this obtained background can be exactly solved. Actually, the string theory in this Liouville PP-wave background in light-cone gauge can be described by the level N SU(2) WZW model plus a Liouville field and four free scalars. The Liouville PP-wave background in string theory has been considered in [15, 16] as a background in which the string theory can be exactly solved. It would be interesting to analyze the string theory in the background (5.17).

5.4 Supergravity duals of disc theories

In this section we will consider a nonlocal theory where the parameter of nonlocality is a tensor. This theory can be naturally defined as the worldvolume theory of M5-branes in the presence of a 3-form C-field with two legs along the worldvolume and one leg transverse to it.[3] To find the corresponding supergravity solution we can start from a $D4$ brane solution and then lift it to 11-dimensional supergravity and send the radius of the 11^{th} direction to infinity, $R_{11} \to \infty$. In this limit, setting $R_{11} M = \bar{M}$, one finds [42]:

$$ds_{11}^2 = h^{-1/3} \Big[f^{-1/3} \Big(dt^2 - \cdots - dx_3^2 - h\, (dx_4^2 + dx_5^2) \Big) + \\ + f^{2/3} \Big(dr^2 + r^2 d\hat{n}^\top d\hat{n} - h\, \frac{r^4}{l_p^6}\, (\hat{n}^\top \bar{M} d\hat{n})^2 \Big) \Big] \,,$$

[3] We note also that the theory on the NS5-branes studied in the previous section could have tensor nonlocality parameters given by RR n-form field strengths for $n > 3$.

$$\sum_{a=2}^{4} C_{45a} dx^a \sim h \, \frac{r^2}{l_p^6} \, d\hat{n}^\top \bar{M} \hat{n} \,, \tag{5.19}$$

where

$$f \stackrel{\text{def}}{=} 1 + \frac{\pi N l_p^3}{r^3} \,, \qquad h^{-1} \stackrel{\text{def}}{=} 1 + \frac{r^2}{l_p^6} \hat{n}^\top \bar{M}^\top \bar{M} \hat{n} \,. \tag{5.20}$$

The decoupling limit of the theory is defined as a limit where $l_p \to 0$ keeping $u = \frac{r}{l_p^3}$ fixed. In this limit, setting $\bar{M}$ as

$$\bar{M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{L} & 0 & 0 \\ 0 & -\bar{L} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{L} \\ 0 & 0 & 0 & -\bar{L} & 0 \end{pmatrix} , \tag{5.21}$$

to preserve 8 supercharges, the above supergravity solution reads

$$\begin{aligned} l_s^2 ds^2 &= h^{-1/3} \Bigg[\frac{u}{(\pi N)^{1/3}} \left(dt^2 - dx_1^2 - \cdots + dx_3^2 - h \, (dx_4^2 + dx_5^2) \right) - \\ &\quad - \frac{(\pi N)^{2/3}}{u^2} \left(du^2 + u^2 d\Omega_3^2 - h u^4 {L_{\text{eff}}}^2 (a_2 d\theta_2 + a_3 d\theta_3 + a_4 d\theta_4)^2 \right) \Bigg] , \\ \sum_{a=2}^{4} C_{45\theta_a} d\theta_a &= h u^2 L_{\text{eff}} (a_2 d\theta_2 + a_3 d\theta_3 + a_4 d\theta_4) \,, \end{aligned} \tag{5.22}$$

where $h^{-1} = 1 + u^2 {L_{\text{eff}}}^2$ and $\theta_1, \ldots, \theta_4$ are angular coordinates parameterizing the sphere S^4 transverse to the brane and a_i's are given by

$$a_2 \stackrel{\text{def}}{=} \cos\theta_3 \,, \qquad a_3 \stackrel{\text{def}}{=} -\sin\theta_2 \cos\theta_2 \sin\theta_3 \,, \qquad a_4 \stackrel{\text{def}}{=} \sin^2\theta_2 \sin^2\theta_3 \,. \tag{5.23}$$

The effective "*discpole*" is also defined by

$$2\pi L_{\text{eff}} = 2\pi \bar{L} \sin\theta_1 \,, \tag{5.24}$$

where $\bar{L}$ has dimension of $(\text{length})^2$.

It is also possible to find a *lightlike discpole* theory by making use of a boost in the x_5 direction. Consider a boost in the x_5 direction given by (5.8) and (5.9). In the limit $\gamma \to \infty$ and taking into account the finiteness of the lightlike dipole [requirement (5.10)], one finds

$$\begin{aligned} ds^2 &= \frac{u}{(\pi N)^{1/3}} \left(-dx^+ dx^- + \frac{{\mu_{\text{eff}}}^2}{4} u^2 (dx^+)^2 - dx_1^2 - \cdots - dx_4^2 \right) - \\ &\quad - \frac{(\pi N)^{2/3}}{u^2} \left(du^2 + u^2 d\Omega_4^2 \right) , \\ \sum_{a=2}^{4} C_{+4\theta_a} d\theta_a &= - \, \frac{\mu_{\text{eff}}}{2} \, u^2 (a_2 d\theta_2 + a_3 d\theta_3 + a_4 d\theta_4) \,, \end{aligned} \tag{5.25}$$

where $2\pi\mu_{\text{eff}} = 2\pi e^{\gamma} L_{\text{eff}}$ is the finite θ-dependent magnitude of the lightlike dipole. Note that in the large boost limit we have $L \to 0$ while $e^{\gamma} L$ is kept fixed.

To study the nonlocal structure of the theory it would be useful to study the expectation values of Wilson loops/surfaces in the theory. In the next section we will study the Wilson loops/surfaces in the theories we have studied so far using the corresponding supergravity solutions.

5.5 The nondecoupling of the center $\mathrm{U}(1) \subset \mathrm{U}(N)$

But before we proceed let us discuss the U(1) center of the gauge group. In the local $\mathcal{N} = 4$ Super-Yang-Mills theory the traces of the gauge field, scalars and fermions (viewed as $N \times N$ matrices) decouple from the rest of the lagrangian which describes an $\mathrm{SU}(N)$ gauge theory. This decoupling has also been demonstrated in the supergravity dual [57, 58]. On the other hand, it is well known that in noncommutative $\mathrm{U}(N)$ gauge theories the traces of the fields do not decouple. On the supergravity dual this has been explained in [59] as a consequence of a nonzero 3-form NSNS flux and a term of the form $\int B_2^{RR} \wedge F_3^{NS} \wedge F_5^{RR}$ in the bulk type-IIB supergravity. Here F_3 and F_5 are the 3-form NSNS and 5-form RR field strengths. In the supergravity dual of noncommutative Yang-Mills theory [60, 61] the field strengths are oriented in such a way that turning on a nonzero B_2^{RR} in the direction perpendicular to the noncommutativity costs energy and this is related to the energy of electric fluxes.

What happens in dipole theory? The U(1) gauge field does not decouple but, unlike noncommutative Yang-Mills theory, the dipole theory is well defined for an $\mathrm{SU}(N)$ gauge group. Therefore, the question arises which gauge group does the supergravity dual describe? Here again there is a nonzero F_3^{NS} but it has a component in the direction of the S^5 and therefore if B_2^{RR} has no legs in the direction of the S^5 then $B_2^{RR} \wedge F_3^{NS} \wedge F_5^{RR} = 0$. This suggests that the supergravity dual describes the dipole theory with $\mathrm{SU}(N)$ rather than $\mathrm{U}(N)$ gauge group. This is just as well because the $\mathrm{U}(N)$ dipole theory is probably ill-defined in the UV (see the β-function calculations in [49, 51]).

What about the trace of the scalars and fermions? In the local $\mathcal{N} = 4$ Super-Yang-Mills it was argued in [57] that operators such as $\mathrm{tr}\left\{\Phi^I\right\}$ (using the notation of subsection 4.2), if they existed, would correspond to supergravity fields in $AdS_5 \times S^5$ with conformal dimension $\Delta = 1$ and they would violate the Breitenlohner-Freedman bound $\Delta \geq 2$. As explained there, this bound is related to the fact that if the conformal dimension is too low the corresponding supergravity mode converges too fast near the boundary. The metric (5.3) behaves very differently from $AdS_5 \times S^5$ near the boundary $u \to \infty$ and therefore the results about $AdS_5 \times S^5$ do not apply. In fact (5.3) has strong curvature for large u and the supergravity approximation is not applicable near the boundary.

In the dipole theories the expression $\mathrm{tr}\left\{\Phi^I\right\}$ is not gauge invariant because $\Phi^I(x)$ transforms as a product of a quark and anti-quark fields at two different points. To make $\mathrm{tr}\left\{\Phi^I\right\}$ gauge invariant one needs to add an open Wilson line inside the trace. We will discuss such operators in more detail in section 7.

6. Closed Wilson loops and Wilson surfaces

In this section we use the dual gravity description of nonlocal theories studied in the previous sections to compute the expectation values of Wilson loops for different theories. According to the AdS/CFT correspondence the expectation value of the Wilson loop of the gauge theory can be computed in the dual string theory description by evaluating the partition function of a string whose worldsheet is bounded by the loop [63, 64]. In the supergravity approximation the dominant contribution comes from the minimal two dimensional surface bounded by the loop. The expectation value of the Wilson loop is

$$\langle W(C)\rangle \sim e^{-S}\,, \tag{6.1}$$

where S is the string action evaluated on the minimal surface bounded by the loop C.

6.1 Dipole theory

The Wilson loop of the dipole gauge theories living on the worldvolume of Dp-branes in the presence of nonzero B-field with one leg along the brane has been studied in [42] using the supergravity solution (5.3). When the distance between a quark and an anti-quark is much bigger than their dipole size, their energy is given by

$$E \sim -\left(\frac{g_{\rm YM}^2 N}{l^2}\right)^{1/(5-p)}\left(1 + c_0 {L_{\rm eff}}^2\left(\frac{g_{\rm YM}^2 N}{l^2}\right)^{2/(5-p)} + \cdots\right), \tag{6.2}$$

where l is the $Q\bar{Q}$ separation, c_0 is a numerical constant and $2\pi L_{\rm eff}$ is the effective dipole vector defined in (5.14). The first term in the above expression is what we have in the ordinary gauge theory and the second term can be interpreted as the dipole-dipole interaction. Note that in our computation leading to (6.2) we have kept fixed the angular coordinates θ_i which are involved in the definition of $L_{\rm eff}$ in (5.14). The angular variables are canonically dual to the R-symmetry charge of the quark (and opposite charge of the anti-quark). Keeping the angular variables fixed translates to a fixed dipole electric moment for the quark and anti-quark that is given by $2\pi L_{\rm eff}$.

6.2 Discpole theory

Let us now compute the Wilson surface of the discpole theory described by the supergravity solution (5.22). From the Wilson surface the potential per unit length of two external straight string-like objects can be calculated. To do this we recognize four different membrane configurations which we parameterize as follows

1. $t=\tau, \quad x_4=\sigma_1, \quad x_1=\sigma_2\equiv x, \quad u=u(x)$ at $\theta_1=\theta_0$=constant.
2. $t=\tau, \quad x_1=\sigma_1, \quad x_4=\sigma_2\equiv x, \quad u=u(x)$ at $\theta_1=\theta_0$=constant.
3. $t=\tau, \quad x_1=\sigma_1, \quad x_2=\sigma_2\equiv x, \quad u=u(x)$ at $\theta_1=\theta_0$=constant.
4. $t=\tau, \quad x_4=\sigma_1, \quad x_5=\sigma_2\equiv x, \quad u=u(x)$ at $\theta_1=\theta_0$=constant.

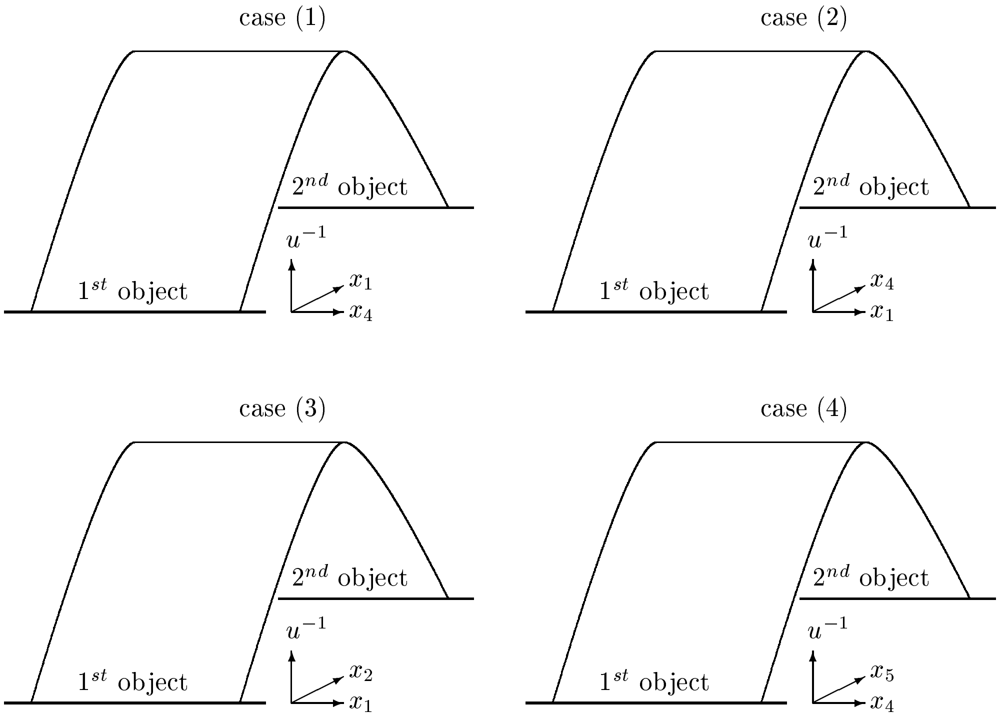

Figure 1: Four different orientations of the membrane in AdS. The nonlocality is in directions x_4, x_5.

Here the membrane worldvolume is parametrized by τ, σ_1 and σ_2. The configurations are depicted in figure 1. We are interested in the action per unit $\Delta\sigma_1\Delta\tau$ area. We therefore consider a finite piece of the membrane given by $0 \leq \sigma_1 \leq L'$ and $0 \leq \tau \leq T$. In the supergravity approximation the action should be proportional to $L'T$. For the case (1) the membrane action

$$S = \frac{1}{(2\pi)^2 l_p^3} \int d\sigma^3 \sqrt{-\det(G_{\mu\nu}\partial_a X^\mu \partial_b X^\nu)}\,, \tag{6.3}$$

reads

$$S = \frac{TL'}{(2\pi)^2} \int dx \sqrt{\frac{u^3}{R^3} + (\partial_x u)^2} \tag{6.4}$$

which is exactly the same as that considered in [64]. Therefore one finds

$$\frac{E}{L'} \sim -\frac{\pi N}{l^2}\,, \tag{6.5}$$

where l is the distance between two external string-like objects of the theory. This means that when the external objects are perpendicular to the $4-5$ plane (the two nonlocality directions) and the plane that passes through the object intersects the 4-5 plane in a line (the 4^{th} direction) as represented by membrane configuration in case (1) the force between them is is not sensitive to the dipole deformation effect.

For the other cases, the membrane actions read

- Case 2

$$S = \frac{TL'}{(2\pi)^2} \int dx \sqrt{\frac{u^3}{R^3} + (1 + u^2 {L_{\rm eff}}^2)(\partial_x u)^2}\,. \tag{6.6}$$

- Case 3

$$S = \frac{TL'}{(2\pi)^2} \int dx \sqrt{(1 + u^2 {L_{\rm eff}}^2)\left(\frac{u^3}{R^3} + (\partial_x u)^2\right)}\,. \tag{6.7}$$

- Case 4

$$S = \frac{TL'}{(2\pi)^2} \int dx \sqrt{\frac{u^3/R^3}{1 + u^2 {L_{\rm eff}}^2} + (\partial_x u)^2}\,. \tag{6.8}$$

The above formulas use the effective discpole $2\pi L_{\rm eff}$ defined in (5.24). Using the same method as [64] the energy as the function of l for all cases is given by

$$\frac{E}{L'} \sim -\frac{\pi N}{l^2}\left(1 + c_0^i L_{\rm eff}\,\frac{(\pi N)^2}{l^4} \cdots\right), \tag{6.9}$$

where c_0^i, $i = 2, 3, 4$ are some numerical constants which, of course, depend on the case number $(2, 3$ or $4)$ that we are considering. The first term in the above expression is the normal interaction between external object in the (2,0) theory. The second term can be interpreted as the discpole interaction between the objects. So the corresponding object represented by the membrane configuration given by cases 2-4 in addition to the normal $\frac{1}{l^2}$ interaction have discpole-discpole interaction as well. Therefore the corresponding states must have discpole moment.

6.3 Lightlike dipole theory

Now we would like to study the potential of the $Q\bar{Q}$ system for the lightlike dipole theory using the corresponding supergravity solution (5.11). We parameterize the string configuration by $x^+ = \tau, x_1 = \sigma = x, u = u(x)$ for fixed x^-. We also keep fixed the angular coordinates which are involved in the definition of $L_{\rm eff}$. In this parameterization, using the supergravity solution (5.11), the string action

$$S = \frac{1}{2\pi l_s^2} \int d\tau d\sigma \sqrt{-\det\left(G_{\mu\nu}\partial_i X^\mu \partial_j X^\nu\right)} \tag{6.10}$$

reads

$$S = \frac{T}{2\pi}\frac{\mu_{\rm eff}}{2} \int dx\; u \sqrt{\left(\frac{u}{R}\right)^{7-p} + (\partial_x u)^2}\,. \tag{6.11}$$

Here we used the lightlike theory's $\mu_{\rm eff}$ defined in equation (5.15). From the form of the action we find that despite the fact that the theory is non-local, the end-points of the string can be fixed at large u. We note that in the noncommutative gauge theory where we have a non-zero B field with both legs along the brane worldvolume we have a problem fixing the end-points [61], though we could fix it using a moving frame [62].

The action (6.11) is minimized when

$$u \frac{(u/R)^{7-p}}{\sqrt{(u/R)^{7-p} + (\partial_x u)^2}} = u_0 \left(\frac{u_0}{R}\right)^{(7-p)/2}, \tag{6.12}$$

where u_0 is the point where $\partial_x u|_{u_0} = 0$. This equation can be solved for $\partial_u x$, and from that the $Q\bar{Q}$ separation is found to be

$$\frac{l}{2} \stackrel{\text{def}}{=} x(u \to \infty) = \frac{R^{(7-p)/2}}{u_0^{(5-p)/2}} \int_1^\infty \frac{dy}{y^{(7-p)/2}\sqrt{y^{9-p}-1}} \,. \tag{6.13}$$

Using (6.11) we can calculate the energy of the $Q\bar{Q}$ system as follows

$$E = \frac{\mu_{\text{eff}}}{4\pi} u_0^2 \left[\int_1^\infty dy \left(\frac{y^{11-p}}{\sqrt{y^{9-p}-1}} - y \right) - \frac{1}{2} \right] . \tag{6.14}$$

Here we subtracted the infinity coming from mass of the W-boson which corresponds to string stretching all the way to $u = \infty$.

Therefore the energy as a function of l is obtained

$$E \sim -\frac{\mu_{\text{eff}}}{4\pi} \left(\frac{g_{YM}^2 N}{l^2} \right)^{2/(5-p)} \,. \tag{6.15}$$

This means that the objects' interaction with each other is only due to the lightlike dipoles they are carrying.

6.4 Lightlike discpole theory

Now we would like to study the Wilson surface in the discpole theory described by the supergravity solution (5.25). Consider an open membrane solution in the background (5.25) parameterizing as following

$$x^+ = \tau \,, \qquad x_1 = \sigma_1 \,, \qquad x_2 = \sigma_2 \equiv x \,, \qquad u = u(x) \,, \qquad x^- = \text{constant} \,. \tag{6.16}$$

Using the lightlike discpole theory's μ_{eff} defined after equation (5.25) we find that the membrane action for this configuration is given by

$$S = \frac{TL'}{(2\pi)^2} \frac{\mu_{\text{eff}}}{2} \int dx \; u \sqrt{\frac{u^3}{R^3} + (\partial_x u)^2} \,, \tag{6.17}$$

which is minimized when

$$\frac{u^4/R^3}{\sqrt{u^3/R^3 + (\partial_x u)^2}} = \text{constant} = \frac{u_0^{5/2}}{R^{3/2}} \,. \tag{6.18}$$

Here u_0 is the point where $\partial_x u|_{u_0} = 0$. This equation can be solved for $\partial_x u$ and thereby the separation between two external objects is given by

$$\frac{l}{2} \stackrel{\text{def}}{=} x(u \to \infty) = \frac{R^{3/2}}{u_0^{1/2}} \int_1^\infty \frac{dy}{y^{3/2}\sqrt{y^5 - 1}} \,. \tag{6.19}$$

Performing the integral one finds

$$l = \frac{2\Gamma(3/5)}{5\Gamma(1/10)} \sqrt{\frac{\pi R^3}{u_0}} = (0.55\ldots) \; \frac{2R^{3/2}}{u_0^{1/2}} \,. \tag{6.20}$$

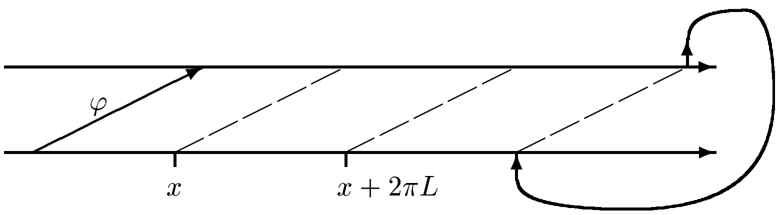

Figure 2: The boundary of the nonlocal dipole theory is a fibration of the x (horizontal) direction over an internal φ (vertical) direction with identification of points separated by $2\pi L$.

Using the expression for the membrane action we can calculate the energy of the system as

$$\frac{E}{L'} = \frac{\mu_{\text{eff}}}{2(2\pi)^2}\, u_0^2 \left[\int_1^\infty dy \left(\frac{y^{7/2}}{\sqrt{y^5 - 1}} - y \right) - \frac{1}{2} \right]. \tag{6.21}$$

Performing the integral one finds

$$\frac{E}{L'} = -(0.14\ldots)\, \frac{\mu_{\text{eff}}}{2(2\pi)^2}\, u_0^2\,. \tag{6.22}$$

From (6.20) and (6.22) we get

$$\frac{E}{L'} = -(0.1\ldots)\, \frac{\mu_{\text{eff}}}{4}\, \frac{N^2}{l^4}\,. \tag{6.23}$$

Similarly to the lightlike dipole theory, this shows that the external objects of the lightlike discpole deformation of the (2,0) theory interact with each other only because of the lightlike discpole moment they are carrying. We note, however, that this is only the effect of the infinite boost. In the undeformed (2,0) theory the string-like objects interact because of their charges, while in the spacelike discpole deformation of the (2,0) theory the objects' interaction has contributions both from the charges and from the discpole moments that they are carrying.

7. Nonlocality in the large N limit

In this section we will examine how the nonlocality of the field theories is manifested in the boundary of the supergravity duals. In the case of the dipole theories we will show that the boundary of the supergravity dual is better viewed as a fibration of $\mathbb{R}^{3,1}$ over an internal space M_5 rather than simply $\mathbb{R}^{3,1} \times S^5$. M_5 is obtain by applying T-duality along 3 internal directions in S^5. In general the fibration has a singular locus but away from that locus the fibration is smooth and has a structure group that is generated by a finite number of translations in $\mathbb{R}^{3,1}$. The translation vectors can be identified with the dipole-vectors of the theory. It means that pairs of points in $\mathbb{R}^{3,1}$ that are separated by an integral product of a dipole-vector are connected by a path through M_5. This is depicted in figure 2 where φ is a compact direction in M_5. This picture was demonstrated in [41] for a special case and we will show this in subsection 7.1 for the general dipole theory.

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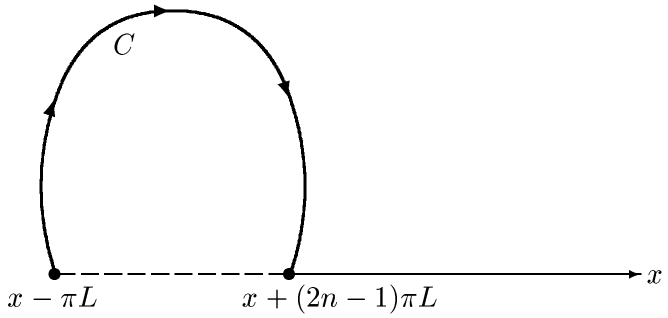

Figure 3: Open Wilson line with R-charge. The opening is a vector in the x direction with length proportional to the R-charge.

Another aspect of the nonlocality of the dipole theories is the existence of *open Wilson lines* depicted in figure 3. Consider a field Φ with dipole vector $2\pi\vec{L}$. Let C be an open path from $\vec{x} + (2n-1)\pi\vec{L}$ to $\vec{x} - \pi\vec{L}$ where $n \in \mathbb{Z}_+$ is a positive integer. We can define the gauge invariant operator

$$W(C) = \mathrm{tr}\left\{Pe^{i\int_C A}\Phi(\vec{x})\Phi(\vec{x}+2\pi\vec{L})\cdots\Phi(\vec{x}+2\pi n\vec{L})\right\}. \tag{7.1}$$

Since $2\pi\vec{L}$ is proportional to the R-charge a generic gauge invariant operator with R-charge must have an open Wilson line with the opening proportional to the R-charge.

Moving on to the generalized nonlocal theories, we conjecture that there are operators with R-charge that correspond to open manifolds. For example, consider the discpole theory which is a nonlocal deformation of the $(2,0)$ theory. The $(2,0)$ theory is believed to have *Wilson surface* operators, that is, operators that correspond to closed surfaces. Consider the nonlocal deformation that assigns an area in the x_4, x_5 plane to R-charge. Such a theory, we conjecture, will have operators that correspond to open surfaces. A typical open-Wilson-surface operator has a boundary that lies in a single plane parallel to the x_4, x_5 directions. Such a surface is depicted in figure 4. By analogy with the open Wilson lines of dipole theories, the area of the opening will be proportional to the R-charge of the operator. Presumably, such operators can have a disconnected boundary $\bigcup_i D_i$ where each D_i is a curve on a plane of constant x_0, x_1, x_2, x_3 and the sum of the (signed) areas bounded by all the D_i's is proportional to the R-charge.

7.1 Nonlocality in dipole theories

We parameterize the matrix of dipole vectors M, defined in subsection 3.1, as the matrix

$$M = \begin{pmatrix} 0 & L_1 & 0 & 0 & 0 & 0 \\ -L_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_2 & 0 & 0 \\ 0 & 0 & -L_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & L_3 \\ 0 & 0 & 0 & 0 & -L_3 & 0 \end{pmatrix}. \tag{7.2}$$

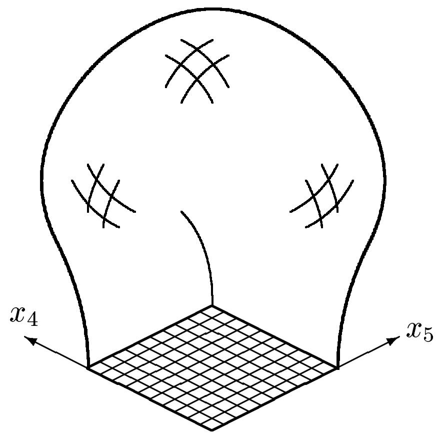

Figure 4: Open Wilson surface with R-charge. The boundary traces out a closed curve in the x_4, x_5 plane that bounds an area proportional to the R-charge. The opening (depicted as a cross hatched square) could be of any shape.

The parameters $2\pi L_1, 2\pi L_2, 2\pi L_3$ are the dipole lengths of the scalar fields. The dipole lengths of the fermionic fields are

$$\lambda_1 \stackrel{\text{def}}{=} \pi(L_1 - L_2 - L_3)\,, \qquad \lambda_2 \stackrel{\text{def}}{=} \pi(L_2 - L_1 - L_3)\,, \qquad \lambda_3 \stackrel{\text{def}}{=} \pi(L_3 - L_1 - L_2)\,,$$
$$\lambda_4 \stackrel{\text{def}}{=} \pi(L_1 + L_2 + L_3) = -(\lambda_1 + \lambda_2 + \lambda_3)\,.$$

To see how all these distances appear as the nonlocality parameters we start with the metric and NSNS B-field in equation (5.3) for $p = 3$. We can parameterize S^5 as a T^3 fibration over S^2, as in [42],

$$\hat{n} = (y_1 \sin\varphi_1,\, y_1 \cos\varphi_1,\, y_2 \sin\varphi_2,\, y_2 \cos\varphi_2,\, y_3 \sin\varphi_3,\, y_3 \cos\varphi_3)\,,$$

with $y_1^2 + y_2^2 + y_3^2 = 1$, where (y_1, y_2, y_3) parameterize the base S^2. The metric and B-field (5.3) can now be written as

$$\begin{aligned}
\alpha'^{-1} ds^2 &= \frac{u^2}{R^2}\left(dt^2 - dx_1^2 - dx_2^2 - \frac{dx_3^2}{1 + u^2 \sum_{i=1}^3 L_i^2 y_i^2}\right) - R^2\left(d\Omega_2^2 + \frac{du^2}{u^2}\right) - \\
&\quad - R^2\left[\sum_{i=1}^3 y_i^2 d\varphi_i^2 - \frac{u^2}{1 + u^2 \sum_{i=1}^3 L_i^2 y_i^2}\left(\sum_{i=1}^3 L_i y_i^2 d\varphi_i\right)^2\right], \\
B &= \frac{u^2}{1 + u^2 \sum_{i=1}^3 L_i^2 y_i^2} \sum_{i=1}^3 L_i y_i^2 d\varphi_i \wedge dx_3\,, \\
e^{2\phi} &= \frac{g_s^2}{1 + u^2 \sum_{i=1}^3 L_i^2 y_i^2}\,.
\end{aligned} \tag{7.3}$$

Here $d\Omega_2^2$ is the metric on the base S^2. For simplicity let us work with lightlike dipole vectors. The background (7.3) is replaced by:

$$
\begin{aligned}
\alpha'^{-1}ds^2 &= \frac{u^2}{R^2}\left[dx^+dx^- - dx_1^2 - dx_2^2 - \frac{u^2}{4}\left(\sum_{i=1}^{3} L_i^2 y_i^2\right)(dx^+)^2\right] - \\
&\quad -R^2\left(\frac{du^2}{u^2} + d\Omega_2^2 + \sum_{i=1}^{3} y_i^2 d\varphi_i^2\right), \\
B &= u^2 \sum_{i=1}^{3} L_i y_i^2 d\varphi_i \wedge dx^+ , \\
e^{2\phi} &= g_s^2 .
\end{aligned}
\tag{7.4}
$$

Next we perform T-duality on the fiber T^3. The metric (7.4) has an isometry $\varphi_i \to \varphi_i + \epsilon_i$ but for each i the isometry $\varphi_i \to \varphi_i + 2\pi$ acts as $(-)^F$. It multiplies the fermion fields by (-1) because the isometry has fixed points near which it acts as a 2π rotation of the tangent plane. Because of the $(-)^F$ T-duality acts somewhat differently than in the usual case. It takes us from type-IIB to type 0A and the T-dual T^3 is smaller by a factor of 2. More precisely, let $P_1, P_2, P_3 \in \mathbb{Z}$ be the momentum generators along the circles parameterized by $\varphi_1, \varphi_2, \varphi_3$. The circles have radii Ry_i. For the purpose of obtaining the exact T-dual background we can double the radii to make them $2Ry_i$ and then restrict to states for which

$$
(-)^F e^{\pi i P_1} = (-)^F e^{\pi i P_2} = (-)^F e^{\pi i P_3} = 1 ,
\tag{7.5}
$$

after T-duality we get type-IIA with the background

$$
\begin{aligned}
\alpha'^{-1}ds^2 &= \frac{u^2}{R^2}\left[dx^+(dx^- - \frac{1}{2}\sum_i L_i d\tilde{\varphi}_i) - dx_1^2 - dx_2^2\right] - \\
&\quad -R^2\left(\frac{du^2}{u^2} + d\Omega_2^2\right) - \frac{1}{4R^2}\sum_{i=1}^{3}\frac{d\tilde{\varphi}_i^2}{y_i^2} .
\end{aligned}
\tag{7.6}
$$

Here $(\tilde{\varphi}_1, \tilde{\varphi}_2, \tilde{\varphi}_3)$ are coordinates on the dual T^3 with the identifications $\tilde{\varphi}_i \sim \tilde{\varphi}_i + 2\pi$, but we have to augment the background with the T-dual of the projection (7.5):

$$
(-)^F e^{\pi i W_1} = (-)^F e^{\pi i W_2} = (-)^F e^{\pi i W_3} = 1 .
\tag{7.7}
$$

Here W_1, W_2, W_3 are string winding numbers. This means that permissible string states have winding numbers that satisfy

$$
W_i - W_j \in 2\mathbb{Z} , \qquad W_i \in \mathbb{Z} , \qquad i,j = 1,2,3 ,
$$

and the state is fermionic if all W_i are odd and bosonic if all W_i are even. From this it follows that the compactification is actually type-0A and the T^3 fibers are the spaces parameterized by $(\tilde{\varphi}_1, \tilde{\varphi}_2, \tilde{\varphi}_3)$ with the identifications

$$
(\tilde{\varphi}_1, \tilde{\varphi}_2, \tilde{\varphi}_3) \sim (\tilde{\varphi}_1 + 2\pi n_1, \tilde{\varphi}_2 + 2\pi n_2, \tilde{\varphi}_3 + 2\pi n_3) , \qquad n_i \in \mathbb{Z}, \qquad n_i - n_j \in 2\mathbb{Z} .
$$

We can define

$$\xi^- \stackrel{\text{def}}{=} x^- - \frac{1}{2}\sum L_i\tilde{\varphi}_i\,, \qquad \xi^+ \stackrel{\text{def}}{=} x^+\,.$$

The metric (7.6) is then simply

$$\alpha'^{-1}ds^2 = \frac{u^2}{R^2}\left[d\xi^- d\xi^+ - dx_1^2 - dx_2^2\right] - R^2\left(\frac{du^2}{u^2} + d\Omega_2^2\right) - \frac{1}{4R^2}\sum_{i=1}^{3}\frac{d\tilde{\varphi}_i^2}{y_i^2}\,. \tag{7.8}$$

It describes the ξ^- direction as fibered over the T^3 in such a way that $\tilde{\varphi}_i \to \tilde{\varphi}_i + 2\pi n_i$ is accompanied by $\xi^- \to \xi^- - \pi\sum_i n_i L_i$, with n_1, n_2, n_3 all odd or all even. The full identification should therefore be

$$\begin{aligned}
&(\tilde{\varphi}_1, \tilde{\varphi}_2, \tilde{\varphi}_3, \xi^+) \sim \left(\tilde{\varphi}_1 + 2\pi n_1, \tilde{\varphi}_2 + 2\pi n_2, \tilde{\varphi}_3 + 2\pi n_3, \xi^+ - \pi\sum_i n_i L_i\right),\\
&n_i \in \mathbb{Z}\,, \qquad n_i - n_j \in 2\mathbb{Z}\,.
\end{aligned} \tag{7.9}$$

It is therefore obvious from the structure of spacetime in the dual supergravity that there are nonlocal interactions between fields at distance $2\pi L_i$ $(i = 1\ldots 3)$ and at distance λ_a $(a = 1\ldots 4)$.

For completeness we will also present the T-dual background to the general background (7.3). It is given by type-0A with metric

$$\begin{aligned}
\alpha'^{-1}ds^2 &= \frac{u^2}{R^2}\left(dx_0^2 - dx_1^2 - dx_2^2\right) - R^2\left(d\Omega_2^2 + \frac{du^2}{u^2}\right) - \\
&\quad - \frac{u^2}{R^2}\left(dx_3 - \frac{1}{2}\sum_{i=1}^{3} L_i d\tilde{\varphi}_i\right)^2 - \frac{1}{R^2}\sum_{i=1}^{3}\frac{d\tilde{\varphi}_i^2}{y_i^2}\\
e^{2\phi} &= \frac{g_s^2}{R^6\prod_{i=1}^3 y_i^2}, \qquad\qquad B = 0\,,
\end{aligned} \tag{7.10}$$

and with the identifications

$$(\tilde{\varphi}_1, \tilde{\varphi}_2, \tilde{\varphi}_3) \sim (\tilde{\varphi}_1 + 2\pi n_1, \tilde{\varphi}_2 + 2\pi n_2, \tilde{\varphi}_3 + 2\pi n_3) \qquad n_i \in \mathbb{Z}, \qquad n_i - n_j \in 2\mathbb{Z}\,. \tag{7.11}$$

7.2 Open Wilson lines in dipole theories

The discussion of Wilson lines is more natural in Euclidean space. We will therefore Wick rotate the background (7.3) to obtain the Euclidean type-IIB background

$$\begin{aligned}
\alpha'^{-1}ds^2 &= \frac{u^2}{R^2}\left(\sum_{\mu=0}^{2} dx_\mu^2 + \frac{dx_3^2}{1 + u^2\sum_{i=1}^3 L_i^2 y_i^2}\right) + R^2\left(d\Omega_2^2 + \frac{du^2}{u^2}\right) + \\
&\quad + R^2\left[\sum_{i=1}^{3} y_i^2 d\varphi_i^2 - \frac{u^2}{1 + u^2\sum_{i=1}^3 L_i^2 y_i^2}\left(\sum_{i=1}^{3} L_i y_i^2 d\varphi_i\right)^2\right],\\
B &= \frac{u^2}{1 + u^2\sum_{i=1}^3 L_i^2 y_i^2}\sum_{i=1}^{3} L_i y_i^2 d\varphi_i \wedge dx_3\,,\\
e^{2\phi} &= \frac{g_s^2}{1 + u^2\sum_{i=1}^3 L_i^2 y_i^2}\,.
\end{aligned} \tag{7.12}$$

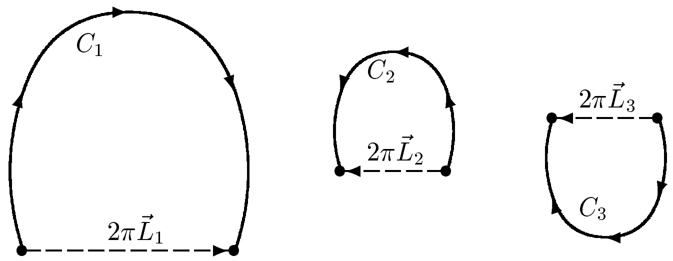

Figure 5: Correlation function of open Wilson lines. It can be nonzero if $\vec{L}_1 + \vec{L}_2 + \vec{L}_3 = 0$.

Similarly, we can Wick rotate the background (7.10) to obtain the T-dual Euclidean type-0A background

$$\begin{aligned}
\alpha'^{-1} ds^2 &= \frac{u^2}{R^2}\sum_{\mu=0}^{2} dx_\mu^2 + R^2\left(d\Omega_2^2 + \frac{du^2}{u^2}\right) + \\
&\quad + \frac{u^2}{R^2}\left(dx_3 - \frac{1}{2}\sum_{i=1}^{3} L_i d\tilde{\varphi}_i\right)^2 + \frac{1}{R^2}\sum_{i=1}^{3}\frac{d\tilde{\varphi}_i^2}{y_i^2} \\
e^{2\phi} &= \frac{g_s^2}{R^6\prod_{i=1}^{3} y_i^2}\,, \qquad B = 0\,.
\end{aligned} \tag{7.13}$$

R-charge is conserved in dipole theories. Unlike a mass deformation of SYM which can classically preserve some R-symmetry but quantum mechanically instanton effects break the classical symmetry, the classical R-symmetry that is preserved by a dipole deformation cannot be broken by instantons. This is because the remaining R-symmetry is a geometrical rotation symmetry in the string theory realization. With the generic dipole vectors given in (7.2) the unbroken symmetry is $\mathrm{U}(1)^3$. Consider now a correlation function $\langle W(C_1)W(C_2)\cdots W(C_r)\rangle$ of open Wilson lines $C_1, C_2, \ldots, C_r$ as in (7.1). Let the openings of the Wilson lines be given by the vectors $2\pi\vec{L}_1, 2\pi\vec{L}_2, \cdots, 2\pi\vec{L}_r$. (See figure 5 for an example.) Because of R-charge conservation we need to have

$$0 = \vec{L}_1 + \vec{L}_2 + \cdots + \vec{L}_r\,.$$

In the supergravity dual (7.13) each open Wilson line corresponds to a closed curve on the boundary. It is constructed from the open curve by closing it through the extra $\tilde{\varphi}_1, \tilde{\varphi}_2, \tilde{\varphi}_3$ directions. Two points P, Q whose x^3 coordinates differ by $2\pi L$ and whose remaining coordinates, including $\tilde{\varphi}_1, \tilde{\varphi}_2, \tilde{\varphi}_3$, are identical can be connected by a microscopic path through the $\tilde{\varphi}_1, \tilde{\varphi}_2, \tilde{\varphi}_3$ dimensions (see figure 6). Using the metric (7.13) we see that the distance between the points P and Q through the $\tilde{\varphi}$-direction along the path $C_{\tilde{\varphi}}$ in figure (6) is of the order of $\frac{1}{R}$ whereas the proper length of a curve between P' and Q with fixed $\tilde{\varphi}_1, \tilde{\varphi}_2, \tilde{\varphi}_3$ is of the order of $\frac{u}{R} \to \infty$ as $u \to \infty$. Thus, the shortcut PQ through the internal $\tilde{\varphi}$ directions has negligible length compared to the rest of the Wilson line.

According to the prescription of [64] we have to complete the closed Wilson loop to a surface into the bulk. In principle, one has to perform a path integral over all worldsheets

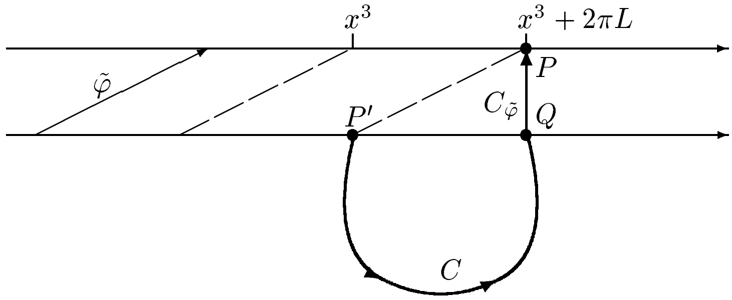

Figure 6: In the supergravity dual, an open Wilson line actually closes through the extra $\tilde{\varphi}$-dimension along the path $C_{\tilde{\varphi}}$. The dashed lines connect points that are identified.

in the bulk with the given boundary conditions but in the case of the local $\mathcal{N}=4$ SYM theory, and for generic, sufficiently smooth Wilson loops the path integral is dominated by a single classical configuration as in [64]. This was also the case in [42] and in section 6 where we studied operators without R-charge.

What would be the analogous prescription for calculating Open Wilson lines in the large N limit of the nonlocal dipole theory? Suppose we are given an open path C on $\mathbb{R}^4$, as the one from P' to Q in figure 6, with the opening proportional to one of the dipole vectors $2\pi\vec{L}_i$. we can close the path C to make it a closed loop by connecting the two endpoints with a straight line through the $\tilde{\varphi}$-direction. Let us denote by $C_{\tilde{\varphi}}$ the extra path that closes the loop. Now that we have a closed loop we can do the path integral over worldsheets in the bulk whose boundary is the closed loop. However, since the metric in the $\tilde{\varphi}$-direction does not have the prefactor $\frac{1}{u^2}$ [see (7.13)] we cannot treat $C_{\tilde{\varphi}}$ as classical. We have to take into account the fluctuations of $C_{\tilde{\varphi}}$, but only in the $\tilde{\varphi}_1,\tilde{\varphi}_2,\tilde{\varphi}_3$ directions and not in the $x_0,\ldots,x_3,y_1,y_2,y_3$ directions. This is because an arbitrarily small deformation of $C_{\tilde{\varphi}}$ in the $x_0,\ldots,x_3$ directions will change the proper length of the path, using the metric (7.13), by terms of order $\frac{u}{R}\to\infty$ on the boundary. This is just as well, since if $C_{\tilde{\varphi}}$ could fluctuate into the $\mathbb{R}^4$ directions the expectation value of the Wilson line would not have depended on the path C which would be absurd! Also, the metric components in the directions of y_1,y_2,y_3 are of the order of R and are assumed large.

Note also that the closed loop $C\bigcup C_{\tilde{\varphi}}$ has nonzero winding number around some of the $\tilde{\varphi}$-directions. This is in accord with the equivalence between winding number and R-symmetry charge. Thus, we have a well-defined prescription for the correlation function of open Wilson lines as in figure 5. To calculate the correlation function in the large N limit we invoke a Born-Oppenheimer approximation where we first fix the $(x_0,\ldots,x_3,y_1,\ldots,y_3,u)$ coordinates of the worldsheet with the appropriate boundary conditions. We then treat $\tilde{\varphi}_1,\tilde{\varphi}_2,\tilde{\varphi}_3$ quantum mechanically and find the quantum partition function of these fields on the string worldsheet. We can take Dirichlet boundary conditions for $\tilde{\varphi}_1,\tilde{\varphi}_2,\tilde{\varphi}_3$ so as to fix the winding numbers in these directions on the boundary.

To leading order in R we can ignore the coordinates $\tilde{\varphi}_1,\tilde{\varphi}_2,\tilde{\varphi}_3$ and consider only the coordinates $x^0,\ldots,x^3,u,y_1,\ldots,y_3$ that describe $AdS_5\times S^2$ (see figure 7). The prescription

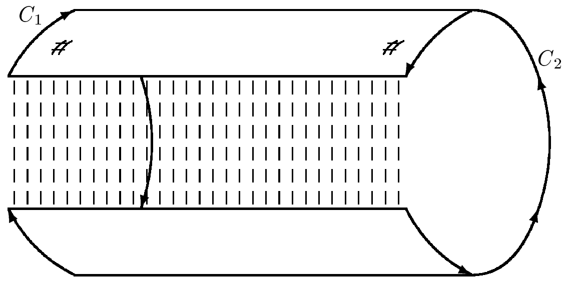

Figure 7: The open worldsheet connecting two open Wilson lines, C_1 and C_2, can be closed with an extra piece of microscopic action (depicted by the dashed pattern).

for calculating the correlation function

$$\langle W(C_1)W(C_2)\cdots W(C_r)\rangle$$

of open Wilson lines $C_1, C_2, \ldots, C_r$ is then as follows.

Consider all possible surfaces $S \subset AdS_5 \times S^2$ such that the intersection of the boundary $u = \infty$ of AdS_5 and S is $C_1 \cup \cdots \cup C_r$. S is also allowed to have a boundary ∂S in the bulk but it is restricted in the following way. For every $0 < u_0 < \infty$ let K_{u_0} be the submanifold $u = u_0$ in $AdS_5 \times S^2$. It has the geometry of $\mathbb{R}^4 \times S^2$. It intersects the boundary of S on a collection of open paths

$$K_{u_0} \cap \partial S = C_1^{(u_0)} \cup C_2^{(u_0)} \cup \cdots \cup C_{m(u_0)}^{(u_0)},$$

where the number $m(u_0)$ of disconnected paths can depend on u_0 because as we vary u_0 paths can split or join. The restriction on S is that for every u_0 the endpoints of each open path $C_i^{(u_0)}$ should be separated by an integer product of the dipole vector $\vec{L}$. If there are several dipole vectors the separation should be a linear combination of the dipole vectors with integer coefficients. We also require that for u_0 small enough $m(u_0) = 0$ which means that all the paths have joined together and therefore $K_{u_0} \cap \partial S$ is either a union of closed loops or the empty set. We now have to find the minimum area of all allowed S's. To leading order in this approximation

$$-\log\langle W(C_1)\cdots W(C_r)\rangle \sim \min \mathrm{Area}(S)\,.$$

Example. As an example we will calculate the correlation function $\langle W(C_1)W(C_2)\rangle$ where C_1 and C_2 are straight segments of length $2\pi L$. To be specific we take

$$\begin{aligned} &W(x_0,x_1,x_2,x_3) \stackrel{\text{def}}{=} \\ &\stackrel{\text{def}}{=} \mathrm{tr}\left\{Pe^{-i\int_{-\pi}^{0} A_3(x_0,x_1,x_2,x_3-tL)dt} Z(x_0,x_1,x_2,x_3) Pe^{i\int_0^{\pi} A_3(x_0,x_1,x_2,x_3+tL)dt}\right\} \end{aligned} \tag{7.14}$$

and we assume that Z is an appropriate (complex) linear combination of the 6 scalars Φ^I with a dipole vector of length $2\pi L$ in the x_3 direction. We will calculate

$$-\log\left\langle W\left(0,-\frac{1}{2}a,0,0\right)^{\dagger} W\left(0,\frac{1}{2}a,0,b\right)\right\rangle.$$

For this purpose we find a surface whose boundary contains the two segments C_1 and C_2 (see figure 8).

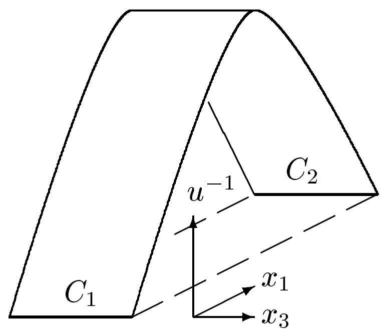

Figure 8: Correlation function of two Open Wilson lines $\langle W(C_1)W(C_2)\rangle$. The strip is an open worldsheet in the u, x_1, x_3 space.

The surface is in the space of $x_0, x_1, x_2, x_3, y_1, y_2, y_3$ but the minimal area of such a surface depends only on the projection onto the space perpendicular to the x_3 direction. We will assume that y_1, y_2, y_3 are constant along S and that $x_0 = x_2 = 0$ along S and we will parameterize $x_3 = y(u)$ so that $y(u_0) = 0$ is the point of S with the smallest value of u and that $y(\infty) = \frac{a}{2}$. Using the metric (7.13) we see that the area of S is then

$$\min \mathrm{Area}(S) = 2L \int_{u_0}^{\infty} \frac{u}{R}\sqrt{y'^2 + \frac{R^4}{u^4}}\, du\,. \tag{7.15}$$

This is exactly the same integral that appears in the calculation of the quark anti-quark potential [63, 64]. After regularization as in [63, 64] we obtain the result

$$-\log\left\langle W\left(0, -\frac{1}{2}a, 0, 0\right)^{\dagger} W\left(0, \frac{1}{2}a, 0, b\right)\right\rangle = \frac{4\pi^2\sqrt{2g_{\mathrm{YM}}^2 N}}{\Gamma(1/4)^4 a} L + \cdots\,,$$

where $(\cdots)$ denotes terms that are subleading when $g_{\mathrm{YM}}^2 N$ is large. We are also assuming $L \gg a$ for the following reason. S is a strip with width $L\sqrt{g_{33}} = \alpha'^{\frac{1}{2}} Lu/R$. At the points of S where u is smallest the proper length of the strip is of the order of

$$\frac{\alpha'^{1/2} L u_0}{R} \sim \frac{\alpha'^{1/2} L}{a}\,,$$

where we used the results of [63, 64] to estimate u_0. If the width of the strip is smaller than $l_s \overset{\mathrm{def}}{=} \alpha'^{1/2}$ we have to include the contribution of the fluctuations in the $\tilde{\varphi}_1, \tilde{\varphi}_2, \tilde{\varphi}_3$ directions which is a boundary effect on the strip S.

When $a \gg L$ we can assume that L is small. For small L the lagrangian of the dipole theory can be written as a small deformation of the lagrangian of $\mathcal{N} = 4$ SYM with gauge group $\mathrm{SU}(N)$ plus 6 free scalar fields and 4 free Dirac fermions. The extra free fields come about because a generic field $Z(x)$ with dipole vector $2\pi\vec{L}$ transforms in the $(N, \overline{N})$ representation of $\mathrm{SU}(N)_{x-\pi\vec{L}} \otimes \mathrm{SU}(N)_{x+\pi\vec{L}}$ where $\mathrm{SU}(N)_x$ is the group at spacetime point x. In the limit $\vec{L} \to 0$ we find that the field Φ transforms in the reducible $(N, \overline{N})$ representation which decomposes into the adjoint representation and a singlet.

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In the limit $a \gg L$ the leading contribution to the open Wilson line $W(x)$ will be the singlet $\mathrm{tr}\{Z\}$ [see (7.14)]. $\mathrm{tr}\{Z\}$ is a free field of conformal dimension 1. We therefore expect

$$-\log\left\langle W\left(0,-\frac{1}{2}a,0,0\right)^{\dagger} W\left(0,\frac{1}{2}a,0,b\right)\right\rangle = 2\log a + \cdots .$$

7.3 Lightcone quantization of strings in the gravity duals of lightlike dipole theories

In order to extend the discussion to the generalized theories, we would like to find the explanation for nonlocality in terms of the original background (7.3). A major simplification occurs in the light-like case (7.4) when the Wilson loops are at constant x^-.

We will use the lightcone formalism for strings in $AdS_5 \times S^5$ as presented in [65]–[67].[4] Equation (21) of [67] describes the lightcone hamiltonian of strings in $AdS_5 \times S^5$ with the metric taken as

$$ds^2 = Y^2 dx^a dx^a + \frac{1}{Y^2} dY^K dY^K, \qquad a = 0\ldots 3, \qquad K = 1\ldots 6.$$

In this subsection we set $2\pi\alpha' = 1$ for convenience. According to [67] the hamiltonian density is

$$\begin{aligned}\mathcal{H} \stackrel{\text{def}}{=} \mathcal{P}^- = \frac{1}{2p^+}(\mathcal{P}_\perp^2 + Y^4 \acute{x}_\perp^2 + Y^4 \mathcal{P}_K \mathcal{P}_K + \acute{Y}^K \acute{Y}^K +\\ + Y^2[p^{+2}(\eta^2)^2 + 2ip^+ \eta\rho^{KN}\eta Y_K \mathcal{P}_N]) -\\ - |Y| Y^K [\eta\rho^K(\acute{\theta} - \sqrt{2}i|Y|\eta\acute{x}) + \text{h.c.}]\,,\end{aligned} \qquad (7.16)$$

where p^+ is the constant lightcone momentum and Y^K $(K = 1\ldots 6)$ are worldsheet fields with associated momenta $\mathcal{P}_K$. The two worldsheet fields $x_\perp$ represent the coordinates x^2, x^3 and their associated momenta are $\mathcal{P}_\perp$. ρ^K are 6-dimensional Dirac matrices and there are two fermions θ and η that are also chiral $SO(6)$ spinors. Their components are denoted by θ_i and η_i $(i = 1\ldots 4)$ with $\theta_i^* = \theta^i$ and $\eta_i^* = \eta^i$. The Dirac matrices ρ^K satisfy $\rho^K_{ij} = -\rho^K_{ji}$ and $\rho^{KN} \stackrel{\text{def}}{=} \rho^{[K}\rho^{\dagger N]}$. The primes over $\acute{\theta}$, $\acute{x}$ and $\acute{Y}$ denote differentiation with respect to the worldsheet coordinate σ. As is standard in lightcone gauge, the coordinate x^- can be found by integrating

$$\acute{x}^- = -\frac{1}{p^+}(\mathcal{P}_\perp \acute{x}_\perp + \mathcal{P}_K \acute{Y}^K) - \frac{i}{2}(\theta^i\acute{\theta}_i + \eta^i\acute{\eta}_i + \theta_i\acute{\theta}^i + \eta_i\acute{\eta}^i)\,. \qquad (7.17)$$

It is easy to extend (7.16) to the gravity dual of the lightlike dipole theory [(5.11) with $p = 3$]. When comparing (5.11) to (7.16) we identify $Y^K = |Y|\hat{n}^K$ and $|Y|^2 = u^2/R^2$. The extra terms to add to (7.16) are the contribution of a $\frac{u^4}{4R^2}(\hat{n}^\top \mathcal{M}^\top \mathcal{M}\hat{n})(dx^+)^2$ term in the metric and an NSNS B-field $\frac{1}{2}u^2 d\hat{n}^\top \mathcal{M}\hat{n}\wedge dx^+$. We obtain the total contribution

$$\mathcal{H}_1 = \frac{p^+}{2}(Y^\top \mathcal{M}^\top \mathcal{M} Y + Y^\top \mathcal{M}\acute{Y}) + (\text{fermions})\,. \qquad (7.18)$$

[4]We wish to thank Andrei Mikhailov for pointing this out.

Here we used the gauge fixing $x^+ = p^+\tau$ and $|Y|^2\sqrt{g}g^{00} = 1$ (see [67]) where (σ,τ) are worldsheet coordinates and g is the worldsheet metric. The expression (7.17) for x^- remains the same.

The extra terms (7.18) can be absorbed by a redefinition

$$\vec{Y} \longrightarrow e^{i\mathcal{M}\sigma}\vec{Y}, \qquad \theta \longrightarrow e^{i\mathcal{M}\sigma}\theta, \qquad \eta \longrightarrow e^{i\mathcal{M}\sigma}\eta\,. \tag{7.19}$$

In the equation for Y, $\mathcal{M}$ should be taken as a 6×6 matrix in the fundamental representation of $so(6)$ and in the equations for θ and η, $\mathcal{M}$ should be taken as a 4×4 matrix in the spinor representation of $so(6)$. After the substitution (7.19) the lightcone hamiltonian has the same form as (7.16) except that the fields Y, θ and η are no longer periodic in σ. Instead

$$\begin{aligned}\vec{Y}(\sigma+p^+,\tau) &= e^{ip^+\mathcal{M}}\vec{Y}(\sigma,\tau)\,,\\ \theta(\sigma+p^+,\tau) &= e^{ip^+\mathcal{M}}\theta(\sigma,\tau)\,, \qquad \eta(\sigma+p^+,\tau) = e^{ip^+\mathcal{M}}\eta(\sigma,\tau).\end{aligned} \tag{7.20}$$

After the redefinition (7.19) the expression (7.17) for x^- becomes:

$$\begin{aligned}\acute{x}^- &= -\frac{1}{p^+}(\mathcal{P}_\perp \acute{x}_\perp + \mathcal{P}_K \acute{Y}^K) - \frac{i}{2}(\theta^i\acute{\theta}_i + \eta^i\acute{\eta}_i + \theta_i\acute{\theta}^i + \eta_i\acute{\eta}^i) - \\ &\quad - \frac{i}{p^+}\mathcal{P}_K\mathcal{M}_{KN}Y^N + \frac{1}{2}(\theta^i\mathcal{M}_i{}^j\theta_j + \eta^i\mathcal{M}_i{}^j\eta_j + \text{c.c.})\,.\end{aligned} \tag{7.21}$$

Note that the second line of (7.21) can be written as $\operatorname{tr}\left\{\mathcal{M}J^0(\sigma)\right\}$ where $(J^0(\sigma), J^1(\sigma))$ are the components of the $so(6)$ R-symmetry worldsheet current. We now define $\tilde{x}^-$ as

$$\tilde{x}^-(\sigma) \stackrel{\text{def}}{=} \int_0^\sigma \left[-\frac{1}{p^+}(\mathcal{P}_\perp \acute{x}_\perp + \mathcal{P}_K\acute{Y}^K) - \frac{i}{2}(\theta^i\acute{\theta}_i + \eta^i\acute{\eta}_i + \theta_i\acute{\theta}^i + \eta_i\acute{\eta}^i)\right]. \tag{7.22}$$

This is the same expression in terms of oscillators as (7.17) but written in terms of the new variables [i.e. after the transformation (7.19)]. The new coordinate $\tilde{x}^-$ is not single valued and satisfies

$$\tilde{x}^-(\sigma+p^+) - \tilde{x}^-(\sigma) = \operatorname{tr}\{\mathcal{M}\mathcal{Q}\}\,, \tag{7.23}$$

where

$$\mathcal{Q}^a \stackrel{\text{def}}{=} \int_0^{p^+}\left[-\frac{i}{p^+}\mathcal{P}^K\tau^a_{KN}Y^N + \frac{1}{2}(\theta^i\tau^a{}_i{}^j\theta_j + \eta^i\tau^a{}_i{}^j\eta_j + \text{c.c.})\right], \qquad a = 1\ldots 15$$

is the $so(6)$ R-symmetry charge. [Here τ^a represents a generator of $so(6)$, τ^a_{KN} is the 6×6 matrix of the generator in the representation 6 of $so(6)$ and $\tau^a{}_i{}^j$ is its 4×4 matrix in the representation 4 of $so(6)$.]

The RHS of (7.23) is exactly the expected dipole length of the corresponding open Wilson operator. The prescription for calculating correlation functions of open Wilson lines in the lightlike dipole theory is as follows. Assume that C_i $(i = 1,2)$ is an open curve in the null plane of x^-, x^1, x^2 and at constant $x^+ \stackrel{\text{def}}{=} x_i^+$. The correlation function $\langle W(C_1)^\dagger W(C_2)\rangle$ can be calculated from the string amplitude to propagate from the string state corresponding to C_1 at $x^+ = x_1^+$ to the string state corresponding to C_2 at $x^+ = x_2^+$.

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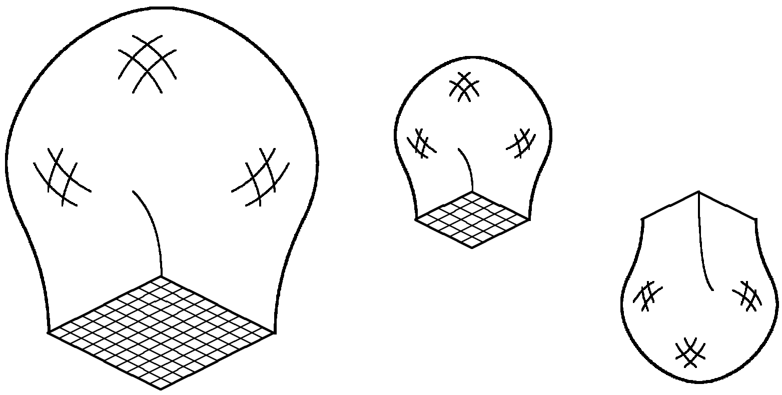

Figure 9: Correlation function of Open Wilson surfaces. For a nonzero value the orientation of the surfaces must be such that the total oriented area of all the openings is zero.

The string state corresponding to C_1 is such that the worldsheet fields $(x_\perp(\sigma), \tilde{x}^-(\sigma))$ trace out C_1 as $0 \le \sigma \le p^+$ and $|Y|^2 \to \infty$. The state is also required to have the specified R-charge corresponding to its dipole vector opening. Note that in this prescription we use the $AdS_5 \times S^5$ hamiltonian $\mathcal{H}$ given in equation (7.16) and not the deformed one $(\mathcal{H}+\mathcal{H}_1)$.

7.4 Open Wilson surfaces in generalized twisted theories

Similarly, we can discuss correlation functions of open Wilson surfaces in discpole theories. Since the total R-charge of an expression $\langle W(S_1)\cdots W(S_r)\rangle$ must be zero, the sum of the areas of the openings in the surfaces must be zero. (Note that the surfaces are oriented and the areas are therefore signed.)

In more general twisted theories we expect to find operators that are parameterized by manifolds with a boundary. For example, in the theories that are obtained by probing the background

$$\mathcal{V}\mathrm{D}_{[1\ldots p]} - 2\pi M_{ij} J_{[ij]} \in \mathbb{Z}\,,$$

given by equations (3.5)–(3.6) (with $p \le 4$), we expect to find operators that are parameterized by $(p+1)$-dimensional manifolds with p-dimensional boundaries that are restricted to be in a p-dimensional hyperplane that is parallel to $x_1, \ldots, x_p$.

7.5 Open Wilson surfaces from the SUGRA dual

In subsection (7.2) we explained the nonlocality of the dipole theory using T-dual variables. However, an attempt to explain the nonlocality of the discpole theories along similar lines fails as we shall now see. Referring to equation (5.22), we can write S^4 as a T^2 fibration over a base S^2 (with a ring of singular fibers),

$$\hat{n} = (y_3\, y_1 \sin\varphi_1,\, y_1 \cos\varphi_1,\, y_2 \sin\varphi_2,\, y_2 \cos\varphi_2)\,,$$

with $y_1^2 + y_2^2 + y_3^2 = 1$, where (y_1, y_2, y_3) parameterize the base S^2. We will take a generic

$$M = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & L_1 & 0 & 0 \\ 0 & -L_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & L_2 \\ 0 & 0 & 0 & -L_2 & 0 \end{pmatrix} . \tag{7.24}$$

The supersymmetric case (5.22) is $L_1 = L_2 = \bar{L}$. The metric and C-field (5.22) can now be rewritten as

$$\begin{aligned} l_p^{-2} ds^2 &= \frac{u}{R^{1/2}} \left(dx_+ dx_- - dx_1^2 - dx_2^2 - dx_3^2 - \frac{dx_4^2 + dx_5^2}{1 + u^2 \sum_{i=1}^2 L_i^2 y_i^2} \right) - R^2 \left(d\Omega_2^2 + \frac{du^2}{u^2} \right) - \\ &\quad - R^2 \left[\sum_{i=1}^2 y_i^2 d\varphi_i^2 - \frac{u^2}{1 + u^2 \sum_{i=1}^2 L_i^2 y_i^2} \left(\sum_{i=1}^2 L_i y_i^2 d\varphi_i \right)^2 \right] , \\ C &= \frac{u^2}{1 + u^2 \sum_{i=1}^2 L_i^2 y_i^2} \sum_{i=1}^2 L_i y_i^2 d\varphi_i \wedge dx_4 \wedge dx_5 , \end{aligned} \tag{7.25}$$

where we defined $R \stackrel{\text{def}}{=} (\pi N)^{\frac{1}{3}}$. Let us make a few observations about this metric. Set

$$\Delta(u, y)^2 \stackrel{\text{def}}{=} 1 + u^2 (L_1^2 y_1^2 + L_2^2 y_2^2) .$$

For fixed y_1, y_2, y_3 the coordinates φ_1, φ_2 describe a T^2 with metric given by

$$ds^2 = \frac{R^2}{\Delta(u, y)^2} \left[(1 + u^2 L_2^2 y_2^2) y_1^2 d\varphi_1^2 + (1 + u^2 L_1^2 y_1^2) y_2^2 d\varphi_2^2 + 2u^2 L_1 L_2 y_1^2 y_2^2 d\varphi_1 d\varphi_2 \right] . \tag{7.26}$$

This T^2 has area and complex structure:

$$\sqrt{\det G} = \left| \frac{R^2 y_1 y_2}{\Delta(u, y)} \right| l_p^2 , \qquad \tau = \frac{1}{1 + u^2 L_2^2 y_2^2} \left(u^2 L_1 L_2 y_2^2 + i \frac{y_2}{y_1} \Delta(u, y) \right) .$$

We are interested in the behavior of the metric near the boundary $u \to \infty$. In that limit for generic y_1, y_2, y_3 (i.e. away from the singular ring) we have $\Delta \to \infty$. Therefore the area of the T^2 shrinks to zero on the boundary and the complex structure τ becomes real. A duality to type-IIB is not helpful because because real τ is a singular limit of type-IIB. The origin of nonlocality therefore remains mysterious!

8. Discussion

We have argued that many field theories that are realized on branes have a rich class of deformations that break Lorentz invariance and locality. Super-Yang-Mills theory has a "dipole deformation" for which the nonlocality is parameterized by vectors, the $(2, 0)$ theory has a "discpole deformation" for which the nonlocality is parameterized by 2-forms and little string theory has various deformations parameterized by vectors, 3-forms, or 5-forms in type-IIB [corresponding to the cases $p = 4, 2, 0$ in (5.5) respectively] and by 2-forms and 4-forms in type-IIA [corresponding to the cases $p = 3, 1$ in (5.5) respectively]. We have studied the large N limit supergravity duals of these deformed field theories and we have seen that they simplify when the deformation parameters are lightlike. We have analyzed Wilson loops and surfaces in the corresponding theories and we calculated the quark anti-quark potentials for the gauge theories and string anti-string tensions for the $(2, 0)$ theory. The contribution of the (vector or tensor) deformation to the potential can be clearly seen as a subleading term in (6.2),(6.9). In the case of a lightlike deformation we were able to isolate the signature of the deformation as the leading contribution to the potential of a certain (quark or stringlike) object anti-object configuration [see equations (6.15) and (6.23)].

The deformed theories also have open Wilson lines and (presumably) open Wilson surfaces. We calculated simple correlation functions of the open Wilson lines. Here again there is a simplification for the lightlike deformations. We showed that the worldsheet lightcone hamiltonian of a string in the supergravity dual of the deformed (dipole) theory is essentially identical to the worldsheet lightcone hamiltonian for (undeformed) $AdS_5 \times S^5$ [see equation (7.16)]. The only difference is in the boundary conditions on the fields [see equations (7.20) and (7.23)]. These modified boundary conditions also explain the nonlocality in the theory and the opening in the Wilson lines. Understanding the nonlocality of the tensor deformations of the $(2, 0)$ and little string theories still remains a challenge!

Acknowledgments

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A. A brief review of dipole theories

For the sake of being self-contained we will present a brief review of dipole theories. We refer the reader to [41, 42, 27] for more details.

The dipole theories are nonlocal gauge field theories that are also not Lorentz invariant. The gauge group can be either SU(N) or U(N). For the U(N) gauge group the field contents the same as that of $\mathcal{N} = 4$ Super-Yang-Mills theory but the lagrangian is different in the following way.

First we need a constant (R-symmetry) $so(6)$-valued space-time vector M. Similarly to the definition of Super-Yang-Mills theory on a noncommutative R^4 [68]-[70], we modify the product of two fields Φ_a, Φ_b at the spacetime point x to

$$(\Phi_a \star \Phi_b)_x \stackrel{\text{def}}{=} e^{i\pi\langle M\cdot\partial_y, \mathcal{Q}_b\rangle - i\pi\langle M\cdot\partial_z, \mathcal{Q}_a\rangle} \left(\Phi_a(y)\Phi_b(z)\right)|_{y=z=x} \,.$$

Here $\mathcal{Q}$ is the operator R-symmetry charge which takes values in $so(6)$. $\langle\cdot,\cdot\rangle$ is the Killing form on $so(6)$ and the $so(6)$-valued product $M \cdot \partial$ denotes the scalar product of M and ∂ as spacetime vectors. The $\star$-product is associative if all 4 $so(6)$-valued components of the spacetime vector M commute. We now replace all products with $\star$-products.

Note that if Φ is a field such that $\langle M, \mathcal{Q}\rangle\Phi = iL\Phi$, with L a constant spacetime vector, we say that Φ has *dipole-vector* $2\pi L$. This is justified by the expression for the covariant derivative

$$(D_\mu\Phi)_x \stackrel{\text{def}}{=} \partial_\mu\Phi(x) - i(A_\mu\star\Phi)_x + i(\Phi\star A_\mu)_x = \partial_\mu\Phi(x) - iA_\mu(x-\pi L)\Phi(x) + i\Phi(x)A_\mu(x+\pi L)\,.$$

From the lagrangian of the $\mathrm{U}(N)$ dipole theory one can obtain the $\mathrm{SU}(N)$ dipole theory by freezing the gauge field corresponding to the $\mathrm{U}(1)$ center of the gauge group. Note, however, that we cannot impose the tracelessness condition on the scalar and spinor field combinations with nonzero dipole vectors. For example, a field $\Phi(x)$ with dipole vector $2\pi\vec{L}$ does not transform in the adjoint representation of the gauge group but rather in the $(N,\overline{N})$ representation of the product group $\mathrm{U}(N)_{x-\pi\vec{L}} \times \mathrm{U}(N)_{x+\pi\vec{L}}$ where $\mathrm{U}(N)_x$ is the gauge group at the spacetime point x.

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